

2026 AIME II

Time limit: 180 minutes

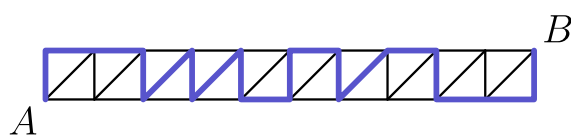
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1. Find the sum of the 10th terms of all arithmetic sequences of integers that have first term equal to 4 and include both 24 and 34 as terms.
2. The figure below shows a grid of 10 squares in a row. Each square has a diagonal connecting its lower left vertex to its upper right vertex. A bug moves along the line segments from vertex to vertex, never traversing the same segment twice and never moving from right to left along a horizontal or diagonal segment. Let N be the number of paths the bug can take from the lower left corner (A) to the upper right corner (B). One such path from A to B is shown by the thick line segments in the figure. Find $\sqrt{N}$.



3. Let $ABCDE$ be a nonconvex pentagon with internal angles $\angle A = \angle E = 90^\circ$ and $\angle B = \angle D = 45^\circ$. Suppose that $DE < AB$, $AE = 20$, $BC = 14\sqrt{2}$, and points B , C , and D lie on the same side of line AE . Suppose further that AB is an integer with $AB < 2026$ and the area of pentagon $ABCDE$ is an integer multiple of 16. Find the number of possible values of AB .
4. For each positive integer n let $f(n)$ be the value of the base-ten numeral n viewed in base b , where b is the least integer greater than the greatest digit in n . For example, if $n = 72$, then $b = 8$, and 72 as a numeral in base 8 equals $7 \cdot 8 + 2 = 58$; therefore $f(72) = 58$. Find the number of positive integers n less than 1000 such that $f(n) = n$.
5. An urn contains n marbles. Each marble is either red or blue, and there are at least 7 marbles of each color. When 7 marbles are drawn randomly from the urn without replacement, the probability that exactly 4 of them are red equals the probability that exactly 5 of them are red. Find the sum of the five least values of n for which this is possible.

6. Find the sum of all real numbers r such that there is at least one point where the circle with radius r centered at $(4, 39)$ is tangent to the parabola with equation $2y = x^2 - 8x + 12$.
7. A standard fair six-sided die is rolled repeatedly. Each time the die reads 1 or 2, Alice gets a coin; each time it reads 3 or 4, Bob gets a coin; and each time it reads 5 or 6, Carol gets a coin. The probability that Alice and Bob each receive at least two coins before Carol receives any coins can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $100m + n$.
8. Isosceles triangle $\triangle ABC$ has $AB = BC$. Let I be the incenter of $\triangle ABC$. The perimeters of $\triangle ABC$ and $\triangle AIC$ are in the ratio $125 : 6$, and all the sides of both triangles have integer lengths. Find the minimum possible value of AB .
9. Let S denote the value of the infinite sum

$$\frac{1}{9} + \frac{1}{99} + \frac{1}{999} + \frac{1}{9999} + \cdots$$

Find the remainder when the greatest integer less than or equal to $10^{100}S$ is divided by 1000.

10. Let $\triangle ABC$ be a triangle with D on $\overline{BC}$ such that $\overline{AD}$ bisects $\angle BAC$. Let ω be the circle that passes through A and is tangent to segment $\overline{BC}$ at D . Let $E \neq A$ and $F \neq A$ be the intersections of ω with segments $\overline{AB}$ and $\overline{AC}$, respectively. Suppose that $AB = 200$, $AC = 225$, and all of AE , AF , BD , and CD are positive integers. Find the greatest possible value of BC .

11. Find the greatest integer n such that the cubic polynomial

$$x^3 - \frac{n}{6}x^2 + (n - 11)x - 400$$

has roots α^2 , β^2 , and γ^2 , where α , β , and γ are complex numbers, and there are exactly seven different possible values for $\alpha + \beta + \gamma$.

12. Consider a tetrahedron with two isosceles triangle faces with side lengths $5\sqrt{10}$, $5\sqrt{10}$, and 10 and two isosceles triangle faces with side lengths $5\sqrt{10}$, $5\sqrt{10}$, and 18. The four vertices of the tetrahedron lie on a sphere with center S , and the four faces of the tetrahedron are tangent to a sphere with center R . The distance RS can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

13. Call finite sets of integers S and T *cousins* if

- S and T have the same number of elements,
- S and T are disjoint, and
- the elements of S can be paired with the elements of T so that the elements in each pair differ by exactly 1.

For example, $\{1, 2, 5\}$ and $\{0, 3, 4\}$ are cousins. Suppose that the set S has exactly 4040 cousins. Find the least number of elements the set S can have.

14. For integers a and b , let $a \circ b = a - b$ if a is odd and b is even, and $a \circ b = a + b$ otherwise. Find the number of sequences $a_1, a_2, a_3, \dots, a_n$ of positive integers such that

$$a_1 + a_2 + a_3 + \cdots + a_n = 12$$

and

$$a_1 \circ a_2 \circ a_3 \circ \cdots \circ a_n = 0,$$

where the operations are performed from left to right; that is, $a_1 \circ a_2 \circ a_3$ means $(a_1 \circ a_2) \circ a_3$.

15. Find the number of ordered 7-tuples $(a_1, a_2, a_3, \dots, a_7)$ having the following properties:

- $a_k \in \{1, 2, 3\}$ for all k .
- $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7$ is a multiple of 3.
- $a_1a_2a_4 + a_2a_3a_5 + a_3a_4a_6 + a_4a_5a_7 + a_5a_6a_1 + a_6a_7a_2 + a_7a_1a_3$ is a multiple of 3.

Solutions: <https://live.poshenloh.com/past-contests/aime/2026II/solutions>

