

2025 AMC 10B Solutions

Typeset by: LIVE by Po-Shen Loh

<https://live.poshenloh.com/past-contests/amc10/2025B/solutions>



Problems © Mathematical Association of America. Reproduced with permission.

1. The instructions on a 350-gram bag of coffee beans say that proper brewing of a large mug of pour-over coffee requires 20 grams of coffee beans. What is the greatest number of properly brewed large mugs of coffee that can be made from the coffee beans in that bag?

A 16

B 17

C 18

D 19

E 20

Solution:

Each mug needs 20 grams, so we divide: $\frac{350}{20} = 17.5$. A half mug isn't a mug, so round down. That leaves 17. Thus, **B** is the correct answer.

2. Jerry wrote down the ones digit of each of the first 2025 positive squares: 1, 4, 9, 6, 5, 6, ... What is the sum of all the numbers Jerry wrote down?

A 9025

B 9070

C 9090

D 9115

E 9160

Solution:

The ones digit of n^2 depends only on the ones digit of n , so the list repeats every 10 terms: 1, 4, 9, 6, 5, 6, 9, 4, 1, 0. One block sums to 45. Now $2025 = 202 \cdot 10 + 5$, so we get 202 full blocks plus the first five terms: $202 \cdot 45 + (1 + 4 + 9 + 6 + 5) = 9090 + 25 = 9115$. Therefore, the answer is **D**.

3. A Pascal-like triangle has 10 as the top row and 10 followed by 1 as the second row. In each subsequent row the first number is 10, the last number is 1, and, as in the standard Pascal's Triangle, each other number in the row is the sum of the two numbers directly above it. The first four rows are shown below.

$$\begin{array}{ccccccc} & & & & 10 & & \\ & & & & & & \\ & & 10 & & 1 & & \\ & & & & & & \\ & 10 & & 11 & & 1 & \\ & & & & & & \\ & 10 & & 21 & & 12 & & 1 \end{array}$$

What is the sum of the digits of the sum of the numbers in the 11th row?

- ☐ A 11
- ☐ B 13
- ☐ C 14
- ☒ D 16
- ☐ E 17

Solution:

Let S_n be the sum of row n . Each entry in row $n - 1$ feeds the two entries just below it, and the fixed border numbers 10 and 1 exactly make up for the terms lost at the edges. So $S_n = 2S_{n-1}$ for $n \geq 3$. With $S_2 = 11$, this gives $S_n = 11 \cdot 2^{n-2}$, so $S_{11} = 11 \cdot 2^9 = 5632$. Its digits sum to $5 + 6 + 3 + 2 = 16$. Thus, **D** is the correct answer.

4. The value of the two-digit number $\underline{a}\underline{b}$ in base seven equals the value of the two-digit number $\underline{b}\underline{a}$ in base nine. What is $a + b$?

- ☒ A 7
- ☐ B 9
- ☐ C 10
- ☐ D 11
- ☐ E 14

Solution:

By place value, $\underline{a}\underline{b}$ in base seven is $7a + b$, and $\underline{b}\underline{a}$ in base nine is $9b + a$. Set them equal: $7a + b = 9b + a$, so $6a = 8b$, that is $3a = 4b$. Thus $a = 4t$ and $b = 3t$.

Because these are two-digit numerals, $t \neq 0$; because a is a base-seven digit, $t \leq 1$. Hence $a = 4$ and $b = 3$, so $a + b = 7$. Therefore, the answer is **A**.

5. In $\triangle ABC$, $AB = 10$, $AC = 18$, and $\angle B = 130^\circ$. Let O be the center of the circle containing points A , B , and C . What is the degree measure of $\angle CAO$?

A 20

B 30

C 40

D 50

E 60

Solution:

Since O is the circumcenter, $OA = OB = OC$. The inscribed angle $\angle B = 130^\circ$ subtends arc AC , and because B is obtuse, the central angle is $\angle AOC = 360^\circ - 2 \cdot 130^\circ = 100^\circ$. Triangle OAC is isosceles, so $\angle CAO = \frac{180^\circ - 100^\circ}{2} = 40^\circ$. (The lengths AB and AC never enter.) Thus, **C** is the correct answer.

6. The line $y = \frac{1}{3}x + 1$ divides the square region defined by $0 \leq x \leq 2$ and $0 \leq y \leq 2$ into an upper region and a lower region. The line $x = a$ divides the lower region into two regions of equal area. Then a can be written as $\sqrt{s} - t$, where s and t are positive integers. What is $s + t$?

A 18

B 19

C 20

D 21

E 22

Solution:

The lower region has area $\int_0^2 \left(\frac{x}{3} + 1\right) dx = \frac{2}{3} + 2 = \frac{8}{3}$. The slice with $0 \leq x \leq a$ is a trapezoid of area $a + \frac{a^2}{6}$. We want that to be half the total, namely $\frac{4}{3}$, so $a^2 + 6a - 8 = 0$ and $a = -3 + \sqrt{17}$. Then $s = 17$, $t = 3$, and $s + t = 20$. Therefore, the answer is **C**.

7. Frances stands 15 meters directly south of a locked gate in a fence that runs east-west. Immediately behind the fence is a box of chocolates, located x meters east of the locked gate. An unlocked gate lies 9 meters east of the box, and another unlocked gate lies 8 meters west of the locked gate. Frances can reach the box by walking toward an unlocked gate, passing through it, and walking toward the box. It happens that the total distance Frances would travel would be the same via either unlocked gate. What is the value of x ?

A $3\frac{2}{7}$

B $3\frac{3}{7}$

C $3\frac{4}{7}$

D $3\frac{5}{7}$

E $3\frac{6}{7}$

Solution:

Put the fence on the x -axis, the locked gate at the origin, and Frances at $(0, -15)$. Then the box is at $(x, 0)$, the east gate at $(x + 9, 0)$, and the west gate at $(-8, 0)$. The east route is $\sqrt{(x + 9)^2 + 15^2} + 9$; the west route is $\sqrt{8^2 + 15^2} + (x + 8) = 17 + x + 8$. Set them equal: $\sqrt{(x + 9)^2 + 225} = x + 16$. Square and simplify to get $18x + 306 = 32x + 256$, so $x = \frac{50}{14} = \frac{25}{7} = 3\frac{4}{7}$. Thus, **C** is the correct answer.

8. Emmy says to Max, "I ordered 36 math club sweatshirts today." Max asks, "How much did each shirt cost?" Emmy responds, "I'll give you a hint. The total cost was $\$ \underline{A} \underline{B} \underline{B} . \underline{B} \underline{A}$, where A and B are digits and $A \neq 0$." After a pause, Max says, "That was a good price." What is $A + B$?

- ☐ A 7
- ☐ B 8
- ☒ C 11
- ☐ D 14
- ☐ E 15

Solution:

In cents the total is $10000A + 1000B + 100B + 10B + A = 10001A + 1110B$. Split evenly among 36 shirts, so it's divisible by 36. Now $10001 \equiv 29$ and $1110 \equiv 30 \pmod{36}$, so we need $29A + 30B \equiv 0$, which reduces to $7A + 6B \equiv 0 \pmod{36}$. The only digit solution with $A \neq 0$ is $A = 6, B = 5$, since $7 \cdot 6 + 6 \cdot 5 = 72$. That's \$655.56, or \$18.21 a shirt, so $A + B = 11$. Therefore, the answer is **C**.

9. How many ordered triples of integers (x, y, z) satisfy the following system of inequalities?

$$-x - y - z \leq -2$$

$$-x + y + z \leq 2$$

$$x - y + z \leq 2$$

$$x + y - z \leq 2$$

A 4

B 8

C 11

D 15

E 17

Solution:

Let $p = -x + y + z$, $q = x - y + z$, $r = x + y - z$. The last three inequalities say $p, q, r \leq 2$, the first says $x + y + z \geq 2$, and $p + q + r = x + y + z$. Since $x = \frac{q+r}{2}$ and so on, p, q, r must all share the same parity. Now count triples with each part ≤ 2 , equal parity, and sum in $[2, 6]$. The even ones are $(2, 2, 2)$, the permutations of $(2, 2, 0)$, of $(2, 0, 0)$, and of $(2, 2, -2)$, giving 10. The only odd one is $(1, 1, 1)$. That's 11 in all, and each yields a unique (x, y, z) . Thus, **C** is the correct answer.

10. Let $f(n) = n^3 - 5n^2 + 2n + 8$, and let $g(n) = n^3 - 6n^2 + 5n + 12$. What is the sum of all integer values of n for which $\frac{f(n)}{g(n)}$ is also an integer?

A 2

B 3

C 4

D 5

E 6

Solution:

Factor both cubics: $f(n) = (n + 1)(n - 2)(n - 4)$ and $g(n) = (n + 1)(n - 3)(n - 4)$. Away from $n \in \{-1, 3, 4\}$, where g vanishes or the ratio is $\frac{0}{0}$, the common factors cancel and $\frac{f(n)}{g(n)} = \frac{n - 2}{n - 3} = 1 + \frac{1}{n - 3}$. That's an integer only when $n - 3 = \pm 1$, so $n = 2$ or $n = 4$. But $n = 4$ kills g , so only $n = 2$ survives, and the sum is 2. Therefore, the answer is **A**.

11. On Monday, 6 students went to the tutoring center at the same time, and each one was randomly assigned to one of the 6 tutors on duty. On Tuesday, the same 6 students showed up, the same 6 tutors were on duty, and the students were again randomly assigned to the tutors. What is the probability that exactly 2 students met with the same tutor both Monday and Tuesday?

A $\frac{1}{16}$

B $\frac{3}{16}$

C $\frac{1}{4}$

D $\frac{3}{8}$

E $\frac{1}{2}$

Solution:

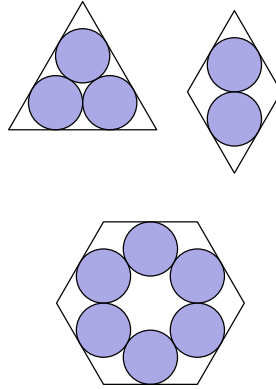
Each day's assignment is a permutation of the 6 students among the 6 tutors.

Comparing the two days, the number who keep the same tutor is the number of fixed points of $\tau = \pi_{\text{Tue}}^{-1} \pi_{\text{Mon}}$, itself a uniformly random permutation of 6 elements. We

want exactly 2 fixed points, so choose those 2 in $\binom{6}{2}$ ways and derange the other 4,

where $D_4 = 9$. The probability is $\frac{\binom{6}{2} D_4}{6!} = \frac{15 \cdot 9}{720} = \frac{3}{16}$. Thus, **B** is the correct answer.

12. The figure below shows an equilateral triangle, a rhombus with a 60° angle, and a regular hexagon, each of them containing some mutually tangent congruent disks. Let T , R , and H , respectively, denote the ratio in each case of the total area of the disks to the area of the enclosing polygon.



Which of the following is true?

A $T = H = R$

B $H < R = T$

C $H = R < T$

D $H < R < T$

E $H < T < R$

Solution:

Take the triangle with side s . Three disks of radius r give $s = 2r(1 + \sqrt{3})$, so $T = \frac{3\pi r^2}{(\frac{\sqrt{3}}{4})s^2} = \frac{(2\sqrt{3} - 3)\pi}{2} \approx 0.73$. For the rhombus with side a , the two disks sit on the

long diagonal $a\sqrt{3} = 6r$, so $r = \frac{a}{2\sqrt{3}}$ and $R = \frac{2\pi r^2}{(\frac{a^2\sqrt{3}}{2})} = \frac{\pi\sqrt{3}}{9} \approx 0.60$. For the

hexagon with side a , each of the six disks touches a side at its midpoint, again giving

$r = \frac{a}{2\sqrt{3}}$ and $H = \frac{6\pi r^2}{(\frac{3\sqrt{3}}{2})a^2} = \frac{\pi\sqrt{3}}{9} \approx 0.60$. So $H = R < T$. Therefore, the answer

is **C**.

13. The altitude to the hypotenuse of a 30° - 60° - 90° right triangle is divided into two segments of lengths $x < y$ by the median to the shortest side of the triangle. What is the ratio $\frac{x}{x+y}$?

A $\frac{3}{7}$

B $\frac{\sqrt{3}}{4}$

C $\frac{4}{9}$

D $\frac{5}{11}$

E $\frac{4\sqrt{3}}{15}$

Solution:

Place the right angle at $C = (0, 0)$, the short leg $CB = 1$ with $B = (1, 0)$, and the long leg $CA = \sqrt{3}$ with $A = (0, \sqrt{3})$. The altitude from C to hypotenuse AB has foot $H = \left(\frac{3}{4}, \frac{\sqrt{3}}{4}\right)$ and runs along $x = \sqrt{3}y$. The median from A to the midpoint $\left(\frac{1}{2}, 0\right)$ of CB meets that altitude at $\left(\frac{3}{7}, \frac{\sqrt{3}}{7}\right)$. This cuts CH (length $\frac{\sqrt{3}}{2}$) into $\frac{4\sqrt{3}}{14}$ and $\frac{3\sqrt{3}}{14}$, so $x = \frac{3\sqrt{3}}{14}$ and $\frac{x}{x+y} = \frac{\frac{3\sqrt{3}}{14}}{\frac{\sqrt{3}}{2}} = \frac{3}{7}$. Thus, **A** is the correct answer.

14. Nine athletes, no two of whom are the same height, try out for the basketball team. One at a time, they draw a wristband at random, without replacement, from a bag containing 3 blue bands, 3 red bands, and 3 green bands. They are divided into a blue group, a red group, and a green group. The tallest member of each group is named the group captain. What is the probability that the group captains are the three tallest athletes?

A $\frac{2}{9}$

B $\frac{2}{7}$

C $\frac{9}{28}$

D $\frac{1}{3}$

E $\frac{3}{8}$

Solution:

The captains are the three tallest exactly when those three land in three different groups, since each is then the tallest of its own group. Drop them into the 9 slots one at a time (3 per group). The second tallest misses the first's group with probability $\frac{6}{8}$, and the third misses both with probability $\frac{3}{7}$. So the probability is $\frac{6}{8} \cdot \frac{3}{7} = \frac{9}{28}$. Therefore, the answer is **C**.

15. The sum

$$\sum_{k=1}^{\infty} \frac{1}{k^3 + 6k^2 + 8k}$$

can be expressed as $\frac{a}{b}$, where a and b are relatively prime positive integers. What is $a + b$?

A 89

B 97

C 102

D 107

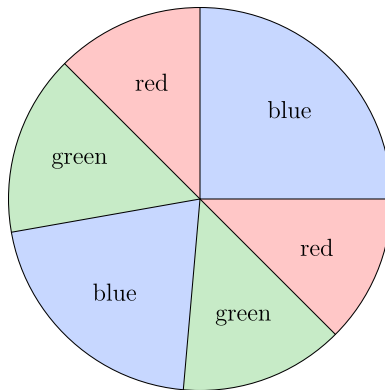
E 129

Solution:

Factor $k^3 + 6k^2 + 8k = k(k + 2)(k + 4)$, then split into partial fractions:

$\frac{1}{k(k + 2)(k + 4)} = \frac{\frac{1}{8}}{k} - \frac{\frac{1}{4}}{k + 2} + \frac{\frac{1}{8}}{k + 4}$. Summing over all k , the coefficient of $\frac{1}{n}$ cancels for $n \geq 5$, so only the first few terms survive: $\frac{1}{8} \left(1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4} \right) = \frac{1}{8} \cdot \frac{11}{12} = \frac{11}{96}$. So $a + b = 11 + 96 = 107$. Thus, **D** is the correct answer.

16. A circle has been divided into 6 sectors of 6 different sizes. Then 2 of the sectors are painted red, 2 painted green, and 2 painted blue so that no two neighboring sectors are painted the same color. One such coloring is shown below.



How many different colorings are possible?

- A 12
- B 16
- C 18
- D 24**
- E 28

Solution:

The six unequal sectors form a fixed cycle of 6 distinguishable positions, so we want proper 3-colorings of a 6-cycle that use each color exactly twice. There are $(3 - 1)^6 + (3 - 1) = 66$ proper 3-colorings altogether. Of these, 6 use only two colors: choose the missing color in 3 ways, then alternate the other two colors in 2 ways. Another 36 have color counts (3, 2, 1): choose the color used three times in 3 ways, choose one of the 2 alternating sets of positions for it, choose the color used twice in 2 ways, and choose its 2 positions among the other 3 in 3 ways. The remaining $66 - 6 - 36 = 24$ use each color exactly twice. Therefore, the answer is **D**.

17. Consider a decreasing sequence of n positive integers $x_1 > x_2 > x_3 > \cdots > x_n$ that satisfies the following conditions. The average (arithmetic mean) of the first 3 terms in the sequence is 2025. For all $4 \leq k \leq n$, the average of the first k terms in the sequence is 1 less than the average of the first $k - 1$ terms in the sequence.

What is the greatest possible value of n ?

A 1013

B 1014

C 1016

D 2016

E 2025

Solution:

Let A_k be the average of the first k terms. Then $A_3 = 2025$ and $A_k = A_{k-1} - 1$ for $k \geq 4$, so $A_k = 2028 - k$. The partial sum is $S_k = k(2028 - k)$, and for $k \geq 4$ the terms are $x_k = S_k - S_{k-1} = 2029 - 2k$, namely $x_4 = 2021, x_5 = 2019, \dots$. These stay positive as long as $2029 - 2k > 0$, that is $k \leq 1014$, with $x_{1014} = 1$. This bound is attainable: take $(x_1, x_2, x_3) = (2030, 2023, 2022)$, which is decreasing, sums to 6075, and lies above $x_4 = 2021$. Thus the greatest possible value is $n = 1014$, so **B** is the correct answer.

18. What is the ones digit of the sum $\lfloor \sqrt{1} \rfloor + \lfloor \sqrt{2} \rfloor + \lfloor \sqrt{3} \rfloor + \cdots + \lfloor \sqrt{2024} \rfloor + \lfloor \sqrt{2025} \rfloor$? (Recall that $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x .)

A 1

B 2

C 3

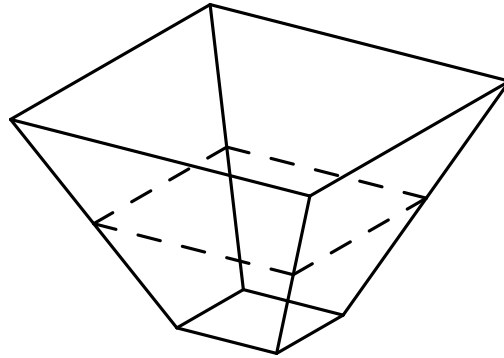
D 5

E 8

Solution:

For each m , $\lfloor \sqrt{n} \rfloor = m$ on the $2m + 1$ integers $m^2 \leq n \leq (m + 1)^2 - 1$. Since $\sqrt{2025} = 45$, the terms with $1 \leq m \leq 44$ contribute $\sum_{m=1}^{44} m(2m + 1)$, and $n = 2025$ tacks on 45. That sum is $\sum_{m=1}^{44} (2m^2 + m) = 2 \cdot \frac{44 \cdot 45 \cdot 89}{6} + \frac{44 \cdot 45}{2} = 58740 + 990 = 59730$, so the total is 59775. Its ones digit is 5. Therefore, the answer is **D**.

19. A container has a 1×1 square bottom, a 3×3 open square top, and four congruent trapezoidal sides, as shown. Starting when the container is empty, a hose that runs water at a constant rate takes 35 minutes to fill the container up to the midline of the trapezoids.



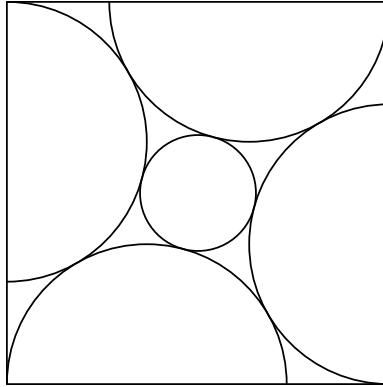
How many more minutes will it take to fill the remainder of the container?

- A 70
- B 85
- C 90
- D 95**
- E 105

Solution:

The container is a square frustum: a horizontal slice at height fraction t has side $1 + 2t$. Extend the sides up to their apex, and the volume out to where the side length is w scales as w^3 . So the whole container is $3^3 - 1^3 = 26$ parts, the piece up to the midline (side 2) is $2^3 - 1^3 = 7$ parts, and the rest is $3^3 - 2^3 = 19$ parts. Those 7 parts take 35 minutes, so each part is 5 minutes. The remaining 19 parts take 95 minutes. Thus, **D** is the correct answer.

20. Four congruent semicircles are inscribed in a square of side length 1 so that their diameters are on the sides of the square, one endpoint of each diameter is at a vertex of the square, and adjacent semicircles are tangent to each other. A small circle centered at the center of the square is tangent to each of the four semicircles, as shown below.



The diameter of the small circle can be written as $(\sqrt{a} + b)(\sqrt{c} + d)$, where a, b, c , and d are integers. What is $a + b + c + d$?

- ☒ A 3
- ☐ B 5
- ☐ C 8
- ☐ D 9
- ☐ E 11

Solution:

Let each semicircle have radius ρ , with centers like $(\rho, 0)$ and $(1, \rho)$. Adjacent semicircles are tangent, so these centers are 2ρ apart: $(1 - \rho)^2 + \rho^2 = 4\rho^2$. This gives $2\rho^2 + 2\rho - 1 = 0$, so $\rho = \frac{\sqrt{3}-1}{2}$. The small circle of radius t sits at $(\frac{1}{2}, \frac{1}{2})$, and it's tangent to a semicircle when its distance to that center equals $\rho + t$. That distance is $\sqrt{2 - \sqrt{3}}$, so $t = \sqrt{2 - \sqrt{3}} - \rho$, and the diameter is $2t = \sqrt{6} - \sqrt{2} - \sqrt{3} + 1 = (\sqrt{3} - 1)(\sqrt{2} - 1)$. So $a + b + c + d = 3 + (-1) + 2 + (-1) = 3$. Therefore, the answer is **A**.

21. Each of the 9 squares in a 3×3 grid is to be colored red, blue, or yellow in such a way that each red square shares an edge with at least one blue square, each blue square shares an edge with at least one yellow square, and each yellow square shares an edge with at least one red square. Colorings that can be obtained from one another by rotations and/or reflections are to be considered the same. How many different colorings are possible?

- A 3
- B 9
- C 12**
- D 18
- E 27

Solution:

First count colorings of a grid whose positions are distinguished. Fix the center square as red and list the four edge-middle colors cyclically. Up to a rotation or reflection, the only possible edge patterns are $YRBR$ and $YRBY$. The first has 4 placements and 3 possible cyclic corner strings, $BRYB$, $BYRB$, and $BYYB$; the second has 8 placements and 2 possible corner strings, $BYBB$ and $BYRB$. Thus there are $4 \cdot 3 + 8 \cdot 2 = 28$ colorings with a red center. The center has 3 possible colors, so there are 84 labeled colorings.

Now apply Burnside's lemma. The identity fixes all 84 colorings. No nonidentity rotation fixes a valid coloring. Each of the two reflections across a horizontal or vertical axis fixes 6 colorings, while each diagonal reflection fixes none. Therefore, the number of colorings up to rotations and reflections is $\frac{84 + 6 + 6}{8} = 12$. Thus, **C** is the correct answer.

22. A seven-digit positive integer is chosen at random. What is the probability that the number is divisible by 11, given that the sum of its digits is 61?

A $\frac{3}{14}$

B $\frac{3}{11}$

C $\frac{2}{7}$

D $\frac{4}{11}$

E $\frac{3}{7}$

Solution:

A digit sum of $61 = 63 - 2$ means all seven digits are 9 except for a total deficit of 2, which gives $\binom{2+6}{6} = 28$ numbers. For divisibility by 11 we need $O - E \equiv 0 \pmod{11}$, where O sums the 4 odd-position digits and E the 3 even ones. Write the deficits as $d_O + d_E = 2$. Then $O - E = 9 - d_O + d_E = 11 - 2d_O$, a multiple of 11 only when $d_O = 0$. So all of the deficit 2 falls on the 3 even positions, giving $\binom{2+2}{2} = 6$ ways. The probability is $\frac{6}{28} = \frac{3}{14}$. Therefore, the answer is **A**.

23. A rectangular grid of squares has 141 rows and 91 columns. Each square has room for two numbers. Horace and Vera each fill in the grid by putting the numbers from 1 through $141 \times 91 = 12,831$ into the squares. Horace fills the grid horizontally: he puts 1 through 91 in order from left to right into row 1, puts 92 through 182 into row 2 in order from left to right, and continues similarly through row 141. Vera fills the grid vertically: she puts 1 through 141 in order from top to bottom into column 1, then 142 through 282 into column 2 in order from top to bottom, and continues similarly through column 91. How many squares get two copies of the same number?

A 7

B 10

C 11

D 12

E 19

Solution:

At row i , column j , Horace writes $91(i - 1) + j$ and Vera writes $141(j - 1) + i$. Set them equal and simplify to get $9i - 14j = -5$, so $i = \frac{14j-5}{9}$, an integer exactly when $j \equiv 1 \pmod{9}$. For $j = 1, 10, 19, \dots, 91$, that's 11 values, and i runs 1, 15, 29, $\dots$, 141, all within range. So 11 squares match. Thus, **C** is the correct answer.

24. A frog hops along the number line according to the following rules. It starts at 0. If it is at 0, then it moves to 1 with probability $\frac{1}{2}$ and it disappears with probability $\frac{1}{2}$. For $n = 1, 2$, or 3 , if it is at n , then it moves to $n + 1$ with probability $\frac{1}{4}$, it moves to $n - 1$ with probability $\frac{1}{4}$, and it disappears with probability $\frac{1}{2}$.

What is the probability that the frog reaches 4?

A $\frac{1}{101}$

B $\frac{1}{100}$

C $\frac{1}{99}$

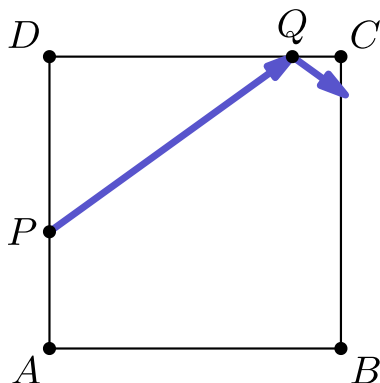
D $\frac{1}{98}$

E $\frac{1}{97}$

Solution:

Let p_n be the probability of reaching 4 from position n , with $p_4 = 1$. The rules give $p_0 = \frac{1}{2}p_1$, $p_1 = \frac{1}{4}p_0 + \frac{1}{4}p_2$, $p_2 = \frac{1}{4}p_1 + \frac{1}{4}p_3$, and $p_3 = \frac{1}{4}p_2 + \frac{1}{4}$. Work upward: $p_1 = \frac{2}{7}p_2$ and $p_2 = \frac{7}{26}p_3$. These unwind to $p_3 = \frac{26}{97}$, $p_2 = \frac{7}{97}$, $p_1 = \frac{2}{97}$, and finally $p_0 = \frac{1}{97}$. Therefore, the answer is **E**.

25. Square $ABCD$ has sides of length 4. Points P and Q lie on $\overline{AD}$ and $\overline{CD}$, respectively, with $AP = \frac{8}{5}$ and $DQ = \frac{10}{3}$. A path begins along the line segment from P to Q and continues by reflecting against the sides of $ABCD$ (with congruent incoming and outgoing angles), as shown in the figure. If the path hits a vertex of the square, then it terminates there; otherwise it continues forever.



At which vertex does the path terminate?

- ☐ A A
- ☒ B B
- ☐ C C
- ☐ D D
- ☐ E The path continues forever.

Solution:

Place $A = (0, 0)$, $B = (4, 0)$, $C = (4, 4)$, $D = (0, 4)$, so $P = (0, \frac{8}{5})$ and $Q = (\frac{10}{3}, 4)$. The initial direction is $(\frac{10}{3}, \frac{12}{5}) \parallel (25, 18)$. Unfold the billiard into a grid of reflected copies and follow the straight line from P . At a grid corner $(4a, 4b)$, the horizontal coordinate gives $t = \frac{4a}{25}$. The vertical coordinate then requires $10 + 18a = 25b$, so $a \equiv 5 \pmod{25}$. The first positive possibility is $a = 5$, giving $b = 4$ and the unfolded corner $(20, 16)$. Crossing 5 cells across (odd) puts it on the side $x = 4$, and 4 cells up (even) puts it on $y = 0$. That's vertex $(4, 0) = B$. Thus, **B** is the correct answer.

Problems: <https://live.poshenloh.com/past-contests/amc10/2025B>

