2009 AMC 12A

Time limit: 75 minutes

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https://live.poshenloh.com/past-contests/amc12/2009A

1. Kim's flight took off from Newark at $10{:}34$ am and landed in Miami at $1{:}18$ pm. Both cities are in the same time zone. If her flight took h hours and m minutes, with $0 \leq m < 60$, what is $h + m$?

A 46

B 47

C 50

D 53

E 54

2. Which of the following is equal to

$$1+\frac{1}{1+\frac{1}{1+1}}?$$

A $\frac{5}{4}$

B $\frac{3}{2}$

C $\frac{5}{3}$

D 2

E 3

3. What number is one third of the way from $\frac{1}{4}$ to $\frac{3}{4}$?

A $\frac{1}{3}$

B $\frac{5}{12}$

C $\frac{1}{2}$

D $\frac{7}{12}$

E $\frac{2}{3}$

4. Four coins are picked out of a piggy bank that contains a collection of pennies, nickels, dimes, and quarters. Which of the following could not be the total value of the four coins, in cents?

A 15

B 25

C 35

D 45

E 55

5. One dimension of a cube is increased by 1, another is decreased by 1, and the third is left unchanged. The volume of the new rectangular solid is 5 less than that of the cube. What was the volume of the cube?

A 8

B 27

C 64

D 125

E 216

6. Suppose that $P = 2^m$ and $Q = 3^n$. Which of the following is equal to 12^{mn} for every pair of integers (m, n)?

A P^2Q

B P^nQ^m

C P^nQ^{2m}

D $P^{2m}Q^n$

E $P^{2n}Q^m$

7. The first three terms of an arithmetic sequence are $2x - 3, 5x - 11$, and $3x + 1$ respectively. The nth term of the sequence is 2009. What is n?

A 255

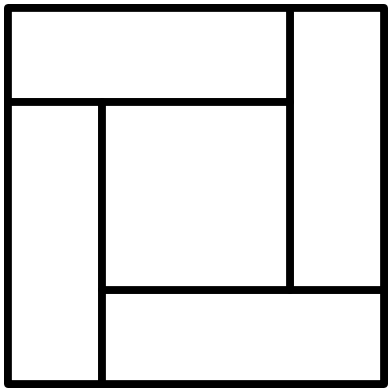
B 502

C 1004

D 1506

E 8037

8. Four congruent rectangles are placed as shown. The area of the outer square is 4 times that of the inner square. What is the ratio of the length of the longer side of each rectangle to the length of its shorter side?



A 3

B $\sqrt{10}$

C $2+\sqrt{2}$

D $2\sqrt{3}$

E 4

9. Suppose that $f(x+3)=3x^2+7x+4$ and $f(x)=ax^2+bx+c$. What is $a+b+c$?

A -1

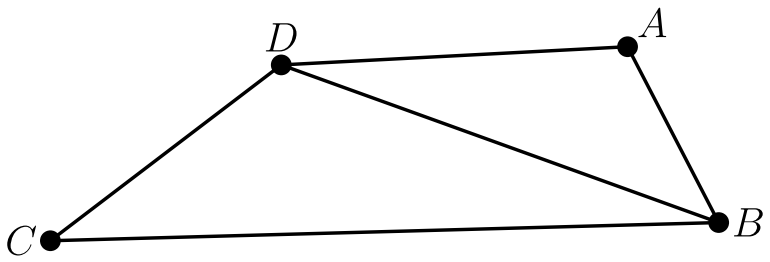
B 0

C 1

D 2

E 3

10. In quadrilateral $ABCD$, $AB = 5, BC = 17, CD = 5, DA = 9$, and BD is an integer. What is BD?



A 11

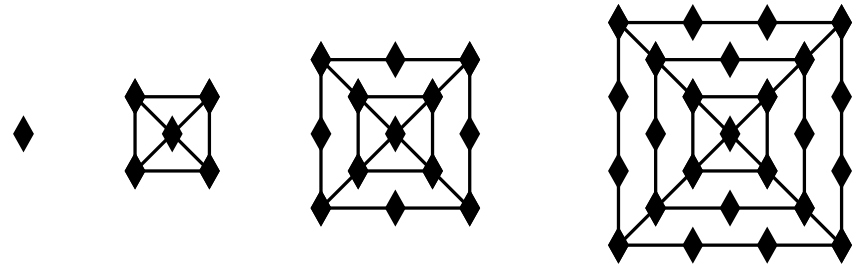
B 12

C 13

D 14

E 15

11. The figures F_1, F_2, F_3, and F_4 shown are the first in a sequence of figures. For $n \geq 3$, F_n is constructed from F_{n-1} by surrounding it with a square and placing one more diamond on each side of the new square than F_{n-1} had on each side of its outside square. For example, figure F_3 has 13 diamonds. How many diamonds are there in figure F_{20}?



F_1 F_2 F_3 F_4

A 401

B 485

C 585

D 626

E 761

12. How many positive integers less than 1000 are 6 times the sum of their digits?

A 0

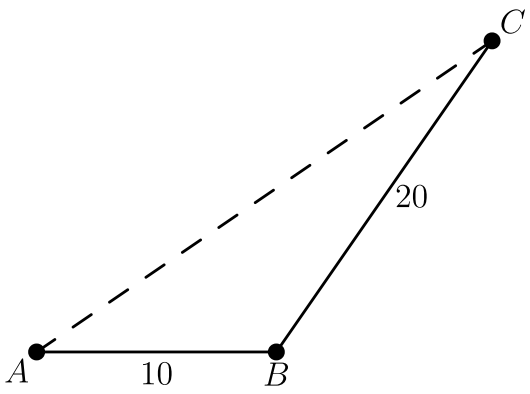
B 1

C 2

D 4

E 12

13. A ship sails 10 miles in a straight line from A to B, turns through an angle between 45° and 60°, and then sails another 20 miles to C. Let AC be measured in miles. Which of the following intervals contains AC^2?



A $[400, 500]$

B $[500, 600]$

C $[600, 700]$

D $[700, 800]$

E $[800, 900]$

14. A triangle has vertices $(0,0), (1,1)$, and $(6m,0)$, and the line $y = mx$ divides the triangle into two triangles of equal area. What is the sum of all possible values of m?

A $-\frac{1}{3}$

B $-\frac{1}{6}$

C $\frac{1}{6}$

D $\frac{1}{3}$

E $\frac{1}{2}$

15. For what value of n is

$$i + 2i^2 + 3i^3 + \cdots + ni^n = 48 + 49i?$$

Note: here $i = \sqrt{-1}$.

A 24

B 48

C 49

D 97

E 98

16. A circle with center C is tangent to the positive x- and y-axes and externally tangent to the circle centered at $(3, 0)$ with radius 1. What is the sum of all possible radii of the circle with center C?

A 3

B 4

C 6

D 8

E 9

17. Let $a + ar_1 + ar_1^2 + ar_1^3 + \cdots$ and $a + ar_2 + ar_2^2 + ar_2^3 + \cdots$ be two different infinite geometric series of positive numbers with the same first term. The sum of the first series is r_1, and the sum of the second series is r_2. What is $r_1 + r_2$?

A 0

B $\frac{1}{2}$

C 1

D $\frac{1+\sqrt{5}}{2}$

E 2

18. For $k > 0$, let $I_k = 10\ldots064$, where there are k zeros between the 1 and the 6. Let $N(k)$ be the number of factors of 2 in the prime factorization of I_k. What is the maximum value of $N(k)$?

A 6

B 7

C 8

D 9

E 10

19. Andrea inscribed a circle inside a regular pentagon, circumscribed a circle around the pentagon, and calculated the area of the region between the two circles. Bethany did the same with a regular heptagon (7 sides). The areas of the two regions were A and B, respectively. Each polygon had a side length of 2. Which of the following is true?

A $A = \frac{25}{49}B$

B $A = \frac{5}{7}B$

C $A = B$

D $A = \frac{7}{5}B$

E $A = \frac{49}{25}B$

20. Convex quadrilateral $ABCD$ has $AB = 9$ and $CD = 12$. Diagonals AC and BD intersect at E, $AC = 14$, and $\triangle AED$ and $\triangle BEC$ have equal areas. What is AE?

A $\frac{9}{2}$

B $\frac{50}{11}$

C $\frac{21}{4}$

D $\frac{17}{3}$

E 6

21. Let $p(x) = x^3 + ax^2 + bx + c$, where a, b, and c are complex numbers. Suppose that

$$\begin{aligned} &p(2009 + 9002\pi i) \\ &= p(2009) \\ &= p(9002) = 0. \end{aligned}$$

What is the number of nonreal zeros of $x^{12} + ax^8 + bx^4 + c$?

A 4

B 6

C 8

D 10

E 12

22. A regular octahedron has side length 1. A plane parallel to two of its opposite faces cuts the octahedron into two congruent solids. The polygon formed by the intersection of the plane and the octahedron has area $\frac{a\sqrt{b}}{c}$, where a, b, and c are positive integers, a and c are relatively prime, and b is not divisible by the square of any prime. What is $a+b+c$?

A 10

B 11

C 12

D 13

E 14

23. Functions f and g are quadratic, $g(x) = -f(100-x)$, and the graph of g contains the vertex of the graph of f. The four x-intercepts on the two graphs have x-coordinates x_1, x_2, x_3, and x_4, in increasing order, and $x_3 - x_2 = 150$. The value of $x_4 - x_1$ is $m + n\sqrt{p}$, where m, n, and p are positive integers, and p is not divisible by the square of any prime. What is $m+n+p$?

A 602

B 652

C 702

D 752

E 802

24. The tower function of twos is defined recursively as follows: $T(1) = 2$ and $T(n+1) = 2^{T(n)}$ for $n \geq 1$. Let $A = (T(2009))^{T(2009)}$ and $B = (T(2009))^{A}$. What is the largest integer k such that

$$\underbrace{\log_2 \log_2 \log_2 \ldots \log_2 B}_{k}$$

is defined?

A 2009

B 2010

C 2011

D 2012

E 2013

25. The first two terms of a sequence are $a_1 = 1$ and $a_2 = \dfrac{1}{\sqrt{3}}$. For $n \geq 1$,

$$a_{n+2} = \frac{a_n + a_{n+1}}{1 - a_n a_{n+1}}.$$

What is $|a_{2009}|$?

A 0

B $2 - \sqrt{3}$

C $\dfrac{1}{\sqrt{3}}$

D 1

E $2 + \sqrt{3}$

Solutions: https://live.poshenloh.com/past-contests/amc12/2009A/solutions