

1989 AMC 8 Solutions

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1. What is the value of

$$(1 + 11 + 21 + 31 + 41) \\ + (9 + 19 + 29 + 39 + 49)?$$

A 150

B 199

C 200

D 249

E 250

Solution:

Pair the terms so each pair sums to 50: $1 + 49$, $11 + 39$, $21 + 29$, $31 + 19$, and $41 + 9$.

There are 5 such pairs, so the total is $5 \times 50 = 250$.

Thus, the correct answer is **E**.

2. What is the value of

$$\frac{2}{10} + \frac{4}{100} + \frac{6}{1000}?$$

A 0.012

B 0.0246

C 0.12

D 0.246

E 246

Solution:

Each fraction is a decimal in a different place: $\frac{2}{10} = 0.2$, $\frac{4}{100} = 0.04$, and $\frac{6}{1000} = 0.006$.

Adding these gives $0.2 + 0.04 + 0.006 = 0.246$.

Thus, the correct answer is **D**.

3. Which of the following numbers is the largest?

A 0.99

B 0.9099

C 0.9

D 0.909

E 0.9009

Solution:

Write each number with four decimal places: 0.9900, 0.9099, 0.9000, 0.9090, 0.9009.

Comparing from the left, 0.9900 has the largest hundredths digit, so 0.99 is the largest.

Thus, the correct answer is **A**.

4. Estimate to determine which of the following is closest to

$$\frac{401}{0.205}.$$

- ☐ A 0.2
- ☐ B 2
- ☐ C 20
- ☐ D 200
- ☒ E 2000

Solution:

Round the numerator and denominator: $\frac{401}{0.205} \approx \frac{400}{0.2}.$

Since $\frac{400}{0.2} = 2000$, the value is closest to **2000**.

Thus, the correct answer is **E**.

5. What is the value of

$$-15 + 9 \times (6 \div 3)?$$

- ☐ A -48
- ☐ B -12
- ☐ C -3
- ☒ D 3
- ☐ E 12

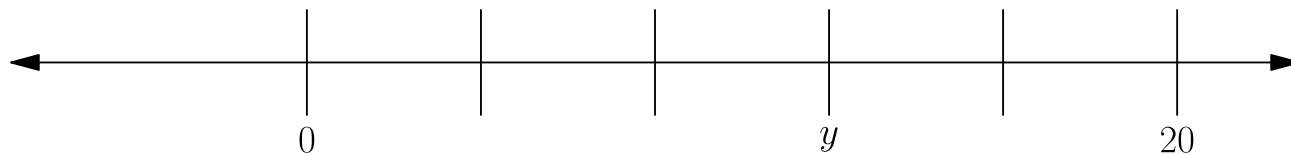
Solution:

Inside the parentheses, $6 \div 3 = 2$. Then multiplication comes before addition, so $9 \times 2 = 18$.

Finally, $-15 + 18 = 3$.

Thus, the correct answer is **D**.

6. If the markings on the number line shown are equally spaced, what is the number y ?



- ☐ A 3
- ☐ B 10
- ☒ C 12
- ☐ D 15
- ☐ E 16

Solution:

From 0 to 20 there are 5 equal spaces, so each space is $\frac{20}{5} = 4$.

The mark labeled y is 3 spaces from 0, so $y = 3 \times 4 = 12$.

Thus, the correct answer is **C**.

7. The value of 20 quarters and 10 dimes equals the value of 10 quarters and n dimes. What is n ?

- ☐ A 10
- ☐ B 20
- ☐ C 30
- ☒ D 35
- ☐ E 45

Solution:

The value of 20 quarters and 10 dimes is $\$5.00 + \$1.00 = \$6.00$.

Ten quarters are worth $\$2.50$, so the n dimes must supply $\$6.00 - \$2.50 = \$3.50$.

That is 35 dimes, so $n = 35$.

Thus, the correct answer is **D**.

8. What is the value of

$$(2 \times 3 \times 4) \left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4} \right)?$$

A 1

B 3

C 9

D 24

E 26

Solution:

Since $2 \times 3 \times 4 = 24$, distribute it over the sum: $24 \cdot \frac{1}{2} + 24 \cdot \frac{1}{3} + 24 \cdot \frac{1}{4}$.

This equals $12 + 8 + 6 = 26$.

Thus, the correct answer is **E**.

9. There are 2 boys for every 3 girls in Ms. Johnson's math class. If there are 30 students in her class, what percent of them are boys?

- A 12%
- B 20%
- C 40%**
- D 60%
- E $66\frac{2}{3}\%$

Solution:

Boys make up 2 out of every $2 + 3 = 5$ students, which is $\frac{2}{5}$ of the class. The total of 30 students is not even needed.

As a percent, $\frac{2}{5} = 40\%$.

Thus, the correct answer is **C**.

10. How many degrees are in the smaller angle between the hour hand and the minute hand of a clock that reads seven o'clock?

A 50°

B 120°

C 135°

D 150°

E 165°

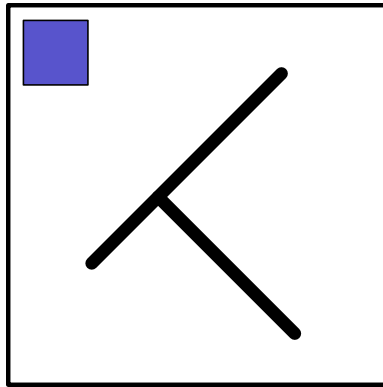
Solution:

The **12** numbers divide the clock into **12** equal sections of $\frac{360^\circ}{12} = 30^\circ$ each.

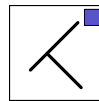
At seven o'clock the hands point to **12** and **7**, which are **5** sections apart the short way, giving $5 \times 30^\circ = 150^\circ$.

Thus, the correct answer is **D**.

11. Which of the five “T-like shapes” would be symmetric to the one shown with respect to the dashed line?



A



B



C



D



E



Solution:

Two figures are symmetric with respect to the dashed line when folding the paper along that line makes one coincide with the other. Reflecting across the vertical dashed line swaps left and right while leaving top and bottom unchanged.

Under this reflection the small square in the top-left corner moves to the top-right corner, the straight crossbar changes its slant from “/” to “\”, and the stem that points down and to the right must instead point down and to the left. Only one choice has all three of these features.

Thus, the correct answer is **B**.

12. What is the value of

$$\frac{1 - \frac{1}{3}}{1 - \frac{1}{2}}?$$

A $\frac{1}{3}$

B $\frac{2}{3}$

C $\frac{3}{4}$

D $\frac{3}{2}$

E $\frac{4}{3}$

Solution:

The numerator is $1 - \frac{1}{3} = \frac{2}{3}$, and the denominator is $1 - \frac{1}{2} = \frac{1}{2}$.

Dividing gives $\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \times 2 = \frac{4}{3}$.

Thus, the correct answer is **E**.

13. Which of the following is equal to

$$\frac{9}{7 \times 53}?$$

A $\frac{0.9}{0.7 \times 53}$

B $\frac{0.9}{0.7 \times 0.53}$

C $\frac{0.9}{0.7 \times 5.3}$

D $\frac{0.9}{7 \times 0.53}$

E $\frac{0.09}{0.07 \times 0.53}$

Solution:

To rewrite the numerator 9 as 0.9, divide it by 10. To keep the fraction equal, divide the denominator by 10 too, which means dividing one of its factors by 10.

Dividing the factor 7 by 10 gives $\frac{0.9}{0.7 \times 53}$, which equals the original. Each other choice changes the value by a factor of 10 or 100.

Thus, the correct answer is **A**.

14. Each of the digits 2, 4, 5, 6, 9 is placed in exactly one box of the subtraction problem shown, in which a two-digit number is subtracted from a three-digit number. What is the smallest difference that is possible?

- ☐ A 58
- ☐ B 123
- ☒ C 149
- ☐ D 171
- ☐ E 176

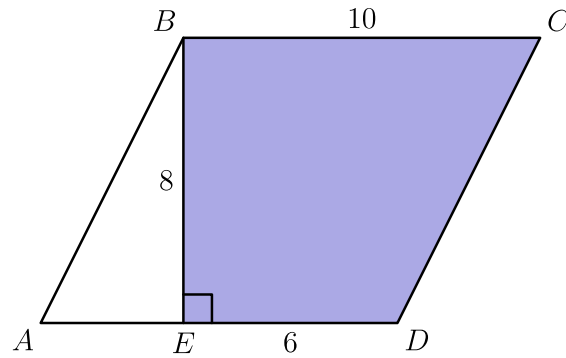
Solution:

The difference is smallest when the three-digit number (the top) is as small as possible and the two-digit number (subtracted) is as large as possible.

The largest two-digit number is 96, using the digits 9 and 6. The smallest three-digit number from the remaining digits 2, 4, 5 is 245. So the smallest difference is $245 - 96 = 149$.

Thus, the correct answer is **C**.

15. In parallelogram $ABCD$ shown, what is the area of the shaded region $BEDC$?



A 24

B 48

C 60

D 64

E 80

Solution:

The parallelogram has base 10 and height 8, so its area is $10 \times 8 = 80$.

The unshaded triangle ABE has base $AE = AD - ED = 10 - 6 = 4$ and height 8, so its area is $\frac{1}{2} \times 4 \times 8 = 16$. The shaded area is $80 - 16 = 64$.

Thus, the correct answer is **D**.

16. In how many ways can 47 be written as the sum of two primes?

A 0

B 1

C 2

D 3

E more than 3

Solution:

Since 47 is odd, a sum of two primes equal to 47 needs one even prime and one odd prime. The only even prime is 2.

That would require the other number to be $47 - 2 = 45$, but $45 = 9 \times 5$ is not prime. So there is no way.

Thus, the correct answer is **A**.

17. The number N is between 9 and 17. Which of the following could be the average of 6, 10, and N ?

A 8

B 10

C 12

D 14

E 16

Solution:

The average is $\frac{6+10+N}{3} = \frac{16+N}{3}$. When $N = 9$ this is $\frac{25}{3} \approx 8.3$, and when $N = 17$ it is 11.

So the average lies strictly between about 8.3 and 11. Among the choices, only 10 falls in this range.

Thus, the correct answer is **B**.

18. A calculator has a reciprocal key that replaces the number currently displayed with its reciprocal. For example, if the display shows 4 and the key is pressed, the display becomes 0.25. If 32 is currently displayed, what is the fewest number of times the reciprocal key must be pressed so that the display again reads 32?

- ☐ A 1
- ☒ B 2
- ☐ C 3
- ☐ D 4
- ☐ E 5

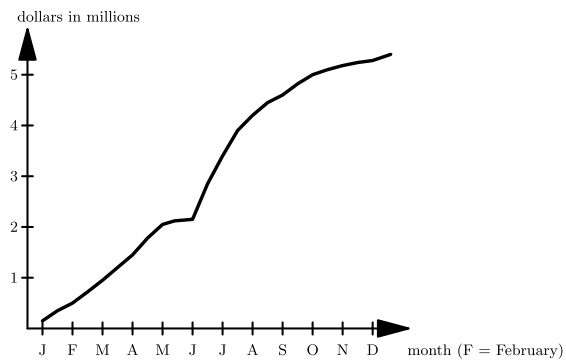
Solution:

Pressing the key once changes 32 to its reciprocal $\frac{1}{32}$.

Pressing it a second time takes the reciprocal again, returning to $\frac{1}{\frac{1}{32}} = 32$. So 2 presses are enough.

Thus, the correct answer is **B**.

19. The graph below shows the total accumulated dollars (in millions) spent by the Surf City government during 1988. For example, about 0.5 million had been spent by the beginning of February and approximately 2 million by the end of April. Approximately how many millions of dollars were spent during the summer months of June, July, and August?



A 1.5

B 2.5

C 3.5

D 4.5

E 5.5

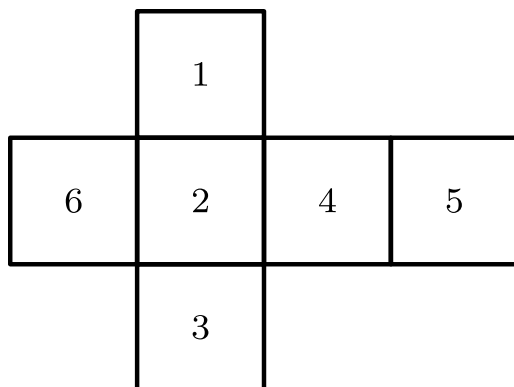
Solution:

The graph gives the total accumulated spending, so the amount spent during June, July, and August equals the accumulated total at the end of August minus the accumulated total at the beginning of June.

The curve is at a bit more than 2 million at the beginning of June and a bit more than 4.5 million by the end of August. The difference is about $4.5 - 2 = 2.5$ million.

Thus, the correct answer is **B**.

20. The figure shown may be folded along the lines to form a number cube. Three faces come together at each corner of the cube. What is the largest sum of three numbers whose faces come together at a corner?



- A 11
- B 12
- C 13
- D 14**
- E 15

Solution:

When the net is folded, the pairs of opposite faces are 1 and 3, 2 and 5, and 4 and 6. Three faces meeting at a corner must come one from each opposite pair.

To maximize the sum, take the larger number from each pair: 3, 5, and 6. These three faces do meet at a corner, and their sum is $3 + 5 + 6 = 14$.

Thus, the correct answer is **D**.

21. Jack had a bag of 128 apples. He sold 25% of them to Jill. Next he sold 25% of those remaining to June. Of those apples still in his bag, he gave the shiniest one to his teacher. How many apples did Jack have then?

A 7

B 63

C 65

D 71

E 111

Solution:

After selling 25% to Jill, Jack keeps $\frac{3}{4} \times 128 = 96$ apples. After selling 25% of those to June, he keeps $\frac{3}{4} \times 96 = 72$ apples.

He then gives 1 to his teacher, leaving $72 - 1 = 71$.

Thus, the correct answer is **D**.

22. The letters A, J, H, S, M, E and the digits $1, 9, 8, 9$ are each cycled separately (shifted one place at a time) to build a numbered list. Starting from $AJHSME\ 1989$, the list begins: line 1 is $JHSMEA\ 9891$, line 2 is $HSMEAJ\ 8919$, line 3 is $SMEAJH\ 9198$, and so on. On what numbered line will $AJHSME\ 1989$ appear for the first time?

A 6

B 10

C 12

D 18

E 24

Solution:

The six letters cycle back to their original order $AJHSME$ every 6 lines, and the four digits cycle back to 1989 every 4 lines.

Both happen on the same line at the least common multiple of 6 and 4, which is 12. So $AJHSME\ 1989$ first reappears on line 12.

Thus, the correct answer is **C**.

23. An artist stacks 14 cubes, each with edges of 1 meter, into a staircase: a 3×3 block of 9 cubes rests on the ground, a 2×2 block of 4 cubes sits on top of it flush into one back corner, and a single cube sits on top of that at the same corner. She paints the entire exposed surface of the sculpture, meaning every face except those resting on the ground. How many square meters does she paint?

A 21

B 24

C 33

D 37

E 42

Solution:

Because each higher layer is flush into one corner, each of the four vertical sides shows a stepped profile of $3 + 2 + 1 = 6$ exposed square faces, for $4 \times 6 = 24$ side faces.

Viewed from directly above, the top faces cover the full 3×3 footprint, adding 9 more faces. The bottom rests on the ground and is not painted, so the total is $24 + 9 = 33$ square meters.

Thus, the correct answer is **C**.

24. A square piece of paper is folded in half. The folded paper is then cut in half by a straight cut parallel to the fold. This forms three rectangles: one large rectangle and two small ones. What is the ratio of the perimeter of one of the small rectangles to the perimeter of the large rectangle?

A $\frac{1}{2}$

B $\frac{2}{3}$

C $\frac{3}{4}$

D $\frac{4}{5}$

E $\frac{5}{6}$

Solution:

Let the square have side 4. Folding in half makes a 2×4 stack of two layers. Cutting parallel to the fold splits it into a strip containing the fold and a strip that does not.

The strip with the fold unfolds into the large rectangle, 2×4 , with perimeter $2(2 + 4) = 12$. The other strip is two separate small rectangles, each 1×4 , with perimeter $2(1 + 4) = 10$. The ratio is $\frac{10}{12} = \frac{5}{6}$.

Thus, the correct answer is **E**.

25. Two wheels are spun, and each wheel's pointer selects one number. The first wheel is divided into four equal regions numbered 3, 4, 5, and 8; the second wheel is divided into three equal regions numbered 6, 7, and 9. What is the probability that the sum of the two selected numbers is even?

A $\frac{1}{6}$

B $\frac{3}{7}$

C $\frac{1}{2}$

D $\frac{2}{3}$

E $\frac{5}{7}$

Solution:

The sum is even when both numbers are even or both are odd. The first wheel has evens $\{4, 8\}$ and odds $\{3, 5\}$, each with probability $\frac{2}{4} = \frac{1}{2}$. The second wheel has even $\{6\}$ with probability $\frac{1}{3}$ and odds $\{7, 9\}$ with probability $\frac{2}{3}$.

So the probability of an even sum is $\frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{6} + \frac{2}{6} = \frac{1}{2}$.

Thus, the correct answer is **C**.

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