1998 AMC 8 Solutions

Typeset by: LIVE, by Po-Shen Loh
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1. For $x = 7$, which of the following is the smallest?

A $\dfrac{6}{x}$

B $\dfrac{6}{x+1}$

C $\dfrac{6}{x-1}$

D $\dfrac{x}{6}$

E $\dfrac{x+1}{6}$

Solution(s):

If we plug in the values into each answer choice, we get the following:

A: $\dfrac{6}{x} = \dfrac{6}{7}$

B: $\dfrac{6}{x+1} = \dfrac{6}{8}$

C: $\dfrac{6}{x-1} = 1$

D: $\dfrac{x}{6} = \dfrac{7}{6}$

E: $\dfrac{x+1}{6} = \dfrac{4}{3}$

Thus, the correct answer is **B**.

2. If $\begin{array}{c|c} a & b \\ \hline c & d \end{array} = \mathrm{a} \cdot \mathrm{d} - \mathrm{b} \cdot \mathrm{c}$, what is the value of $\begin{array}{c|c} 3 & 4 \\ \hline 1 & 2 \end{array}$?

A -2

B -1

C 0

D 1

E 2

Solution(s):

Plugging into the formula above, we get:

$$3 \cdot 2 - 1 \cdot 4 = 2.$$

Thus, the correct answer is **E**.

3. What is the value of:

$$\frac{\frac{3}{8}+\frac{7}{8}}{\frac{4}{5}}?$$

A 1

B $\dfrac{25}{16}$

C 2

D $\dfrac{43}{20}$

E $\dfrac{47}{16}$

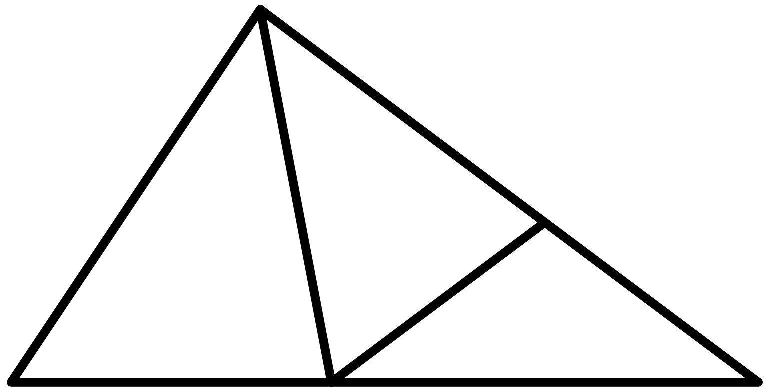
Solution(s):

This evaluates to:

$$\begin{aligned}\frac{\frac{3}{8}+\frac{7}{8}}{\frac{4}{5}} &= \frac{\frac{5}{4}}{\frac{4}{5}} \\ &= \left(\frac{5}{4}\right)^2 \\ &= \frac{25}{16}.\end{aligned}$$

Thus, the correct answer is **B**.

4. How many triangles are in this figure? (Some triangles may overlap other triangles.)



A 9

B 8

C 7

D 6

E 5

Solution(s):

First, we can clearly see the 3 small triangles, and the triangle that encompasses the entire figue. Also, there is 1 more triangle when combining the two rightmost smaller triangles.

This leads to us having 5 triangles.

Thus, the correct answer is **E**.

5. Which of the following numbers is largest?

A 9.12344

B $9.123\overline{4}$

C $9.12\overline{34}$

D $9.1\overline{234}$

E $9.\overline{1234}$

Solution(s):

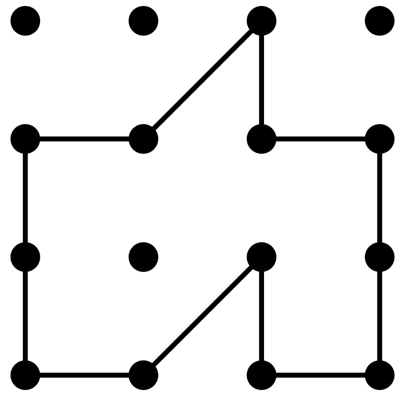
Each of them start with 9.1234. Thus, we need to look at the next digits.

For choices **A** and **B**, the next digit is 4 while it is 3 for **C**, 2 for **D**, and 1 for **E**.

This means we only have to look at **A** and **B** as possible solutions. After the 9.12344, choice **A** terminates while the next term for **B** is 4, so choice **B** is larger.

Thus, the correct answer is **B**.

6. Dots are spaced one unit apart, horizontally and vertically. The number of square units enclosed by the polygon is



A 5

B 6

C 7

D 8

E 9

Solution(s):

Consider the 2×3 rectangle on the bottom. The figure is the same as when we take some area out on the bottom and add the same area on the top. Thus, the area is the same as the the 2×3 rectangle, which is 6

Thus, the correct answer is **B**.

7. $100 \times 19.98 \times 1.998 \times 1000 =$

A $(1.998)^2$

B $(19.98)^2$

C $(199.8)^2$

D $(1998)^2$

E $(19980)^2$

Solution(s):

We will group the first two terms and the last two terms.

This will make the expression equal to

$$(100 \cdot 19.98) \cdot (1.998 \cdot 1000) =$$

$$1998 \cdot 1998 = 1998^2.$$

Thus, the correct answer is **D**.

8. A child's wading pool contains 200 gallons of water. If water evaporates at the rate of 0.5 gallons per day and no other water is added or removed, how many gallons of water will be in the pool after 30 days?

A 140

B 170

C 185

D 198.5

E 199.85

Solution(s):

The amount lost is $0.5 \cdot 30 = 15$ gallons. Therefore, the amount left is

$$200 - 15 = 185.$$

Thus, the correct answer is **C**.

9. For a sale, a store owner reduces the price of a $\$10$ scarf by 20%. Later the price is lowered again, this time by one-half the reduced price. The price is now

A $2.00

B $3.75

C $4.00

D $4.90

E $6.40

Solution(s):

After the 20% reduction, the price is $10\cdot 0.8=$8.

Then, after halving the price, the price is \dfrac {$8}2 = $4.

Thus, the correct answer is **C**.

10. Each of the letters W, X, Y, and Z represents a different integer in the set $\{1, 2, 3, 4\}$, but not necessarily in that order. If

$$\frac{W}{X} - \frac{Y}{Z} = 1,$$

then the sum of W and Y is:

A 3

B 4

C 5

D 6

E 7

Solution(s):

The fractions $\frac{W}{X}$ and $\frac{Y}{Z}$ must be integers as there is no other fractional part that can there twice.

Thus, they are both integers, making 1 a denominator. The only other possible denominator could be 2 with its numerator being 4.

Thus, we could get

$$\frac{3}{1} - \frac{4}{2} = 1,$$

making the sum equal to

$$4 + 3 = 7.$$

Thus, the correct answer is **E**.

11. Harry has 3 sisters and 5 brothers. His sister Harriet has S sisters and B brothers. What is the product of S and B?

A 8

B 10

C 12

D 15

E 18

Solution(s):

Since Harry has 3 sisters and 5 brothers, the family has 3 girls and 6 boys. Then, Harriet would be one or the girls, so she had 2 sisters and 6 brothers. Thus, the product of the number of brothers and sisters is $2 \cdot 6 = 12$.

Thus, the correct answer is **C**.

12. What is the value of the following expression?

$$2\left(1-\frac{1}{2}\right)+3\left(1-\frac{1}{3}\right)+$$
$$4\left(1-\frac{1}{4}\right)+\cdots+10\left(1-\frac{1}{10}\right)$$

A 45

B 49

C 50

D 54

E 55

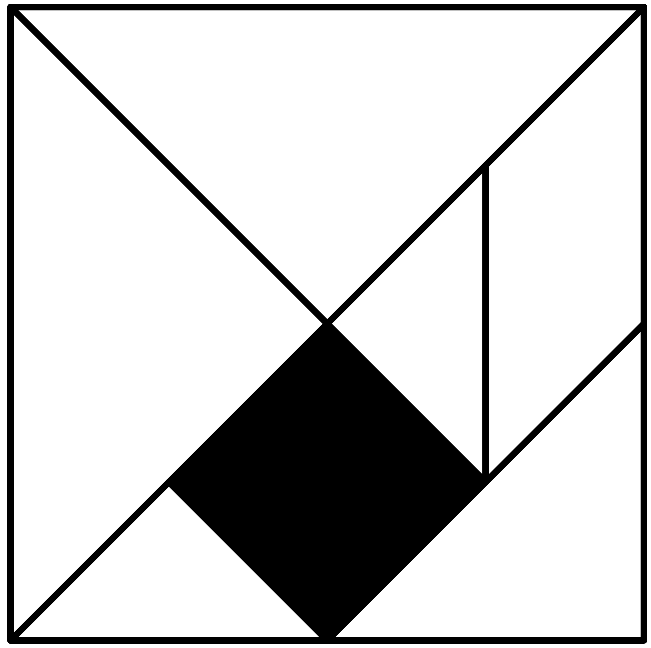
Solution(s):

Directly evaluating, we get that:

$$\begin{aligned}
&2\left(1-\frac{1}{2}\right)+3\left(1-\frac{1}{3}\right)+ \\
&4\left(1-\frac{1}{4}\right)+\cdots+10\left(1-\frac{1}{10}\right) \\
&=(2-1)+\cdots+(10-1) \\
&=1+2\cdots8+9 \\
&=45
\end{aligned}$$

Thus, the correct answer is **A**.

13. What is the ratio of the area of the shaded square to the area of the large square? (The figure is drawn to scale)



A $\frac{1}{6}$

B $\frac{1}{7}$

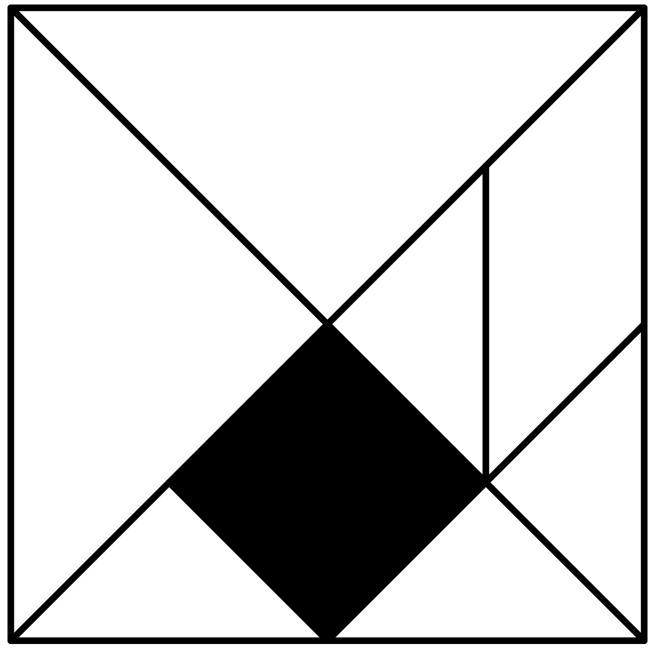
C $\frac{1}{8}$

D $\frac{1}{12}$

E $\frac{1}{16}$

Solution(s):

We could extend the figure to the following:



Then, we could look at the bottom triangle that makes up a quarter of the figure.

Half of that area is the shaded area, so the entire shaded area is $\frac{1}{8}$.

Thus, the correct answer is **C**.

14. An Annville Junior High School, 30% of the students in the Math Club are in the Science Club, and 80% of the students in the Science Club are in the Math Club. There are 15 students in the Science Club. How many students are in the Math Club?

A 12

B 15

C 30

D 36

E 40

Solution(s):

Since 80% of people are in both clubs, the number of people in both clubs is $0.8 \cdot 15 = 12$. This is 30% of the math club, so the number of people in the math club is $\frac{12}{0.3} = 40.$

Thus, the correct answer is **E**.

15. In the very center of the Irenic Sea lie the beautiful Nisos Isles. In 1998 the number of people on these islands is only 200, but the population triples every 25 years. Queen Irene has decreed that there must be at least 1.5 square miles for every person living in the Isles. The total area of the Nisos Isles is 24900 square miles.

Estimate the population of Nisos in the year 2050.

- A 600
- B 800
- C 1000
- D 2000
- E 3000

Solution(s):

The population in 2048, which is 50 years after 1998, is $3^2 \cdot 200 = 1800$.

Therefore, as $2048 \approx 2050$, the population in 2050 is approximately 1800, which is approximately equal to 2000

Thus, the correct answer is **D**.

16. In the very center of the Irenic Sea lie the beautiful Nisos Isles. In 1998 the number of people on these islands is only 200, but the population triples every 25 years. Queen Irene has decreed that there must be at least 1.5 square miles for every person living in the Isles. The total area of the Nisos Isles is 24900 square miles.

Estimate the year in which the population of Nisos will be approximately 6000.

A 2050

B 2075

C 2100

D 2125

E 2150

Solution(s):

This would be the year the population is 30 times as much as in 1998. This means the population triples approximately 3 times, making the year approximately $3 \cdot 25 = 75$ years after 1998. This would be 2073, so 2075 is the best approximation.

Thus, the correct answer is **B**.

17. In the very center of the Irenic Sea lie the beautiful Nisos Isles. In 1998 the number of people on these islands is only 200, but the population triples every 25 years. Queen Irene has decreed that there must be at least 1.5 square miles for every person living in the Isles. The total area of the Nisos Isles is 24900 square miles.

In how many years, approximately, from 1998 will the population of Nisos be as much as Queen Irene has proclaimed that the islands can support?

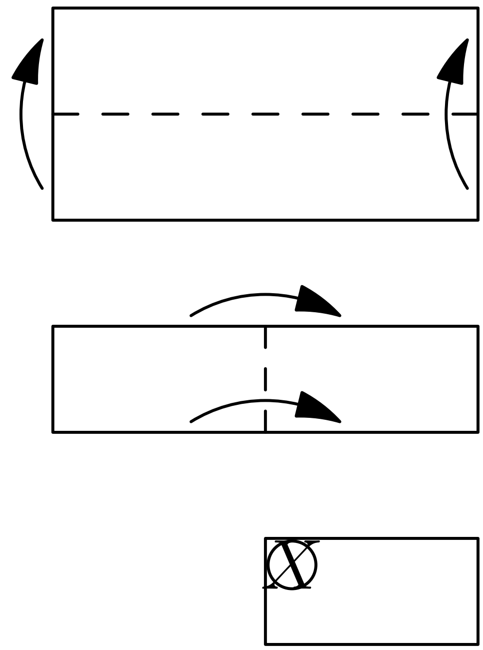
- A 50 years
- B 75 years
- C 100 years
- D 125 years
- E 150 years

Solution(s):

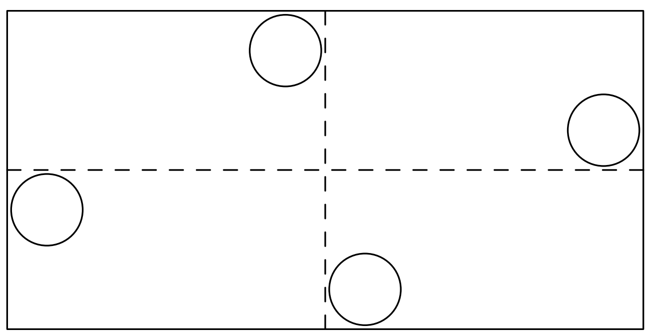
The maximal population is $\dfrac{24900}{1.5} = 16600$. This is 83 times as much as the population in 1998, so it would be about 4 triples from 1998. That would be $25 \cdot 4 = 100$ years.

Thus, the correct answer is **C**.

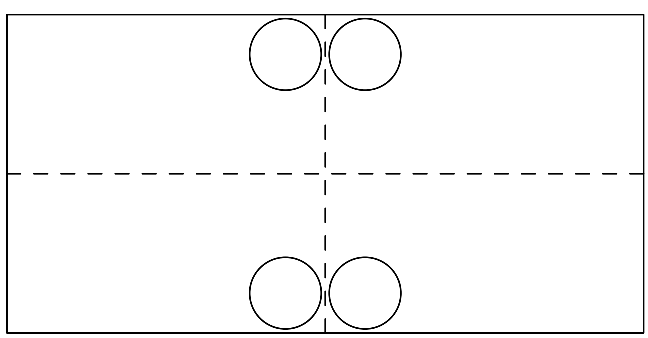
18. As indicated by the diagram below, a rectangular piece of paper is folded bottom to top, then left to right, and finally, a hole is punched at X. What does the paper look like when unfolded?



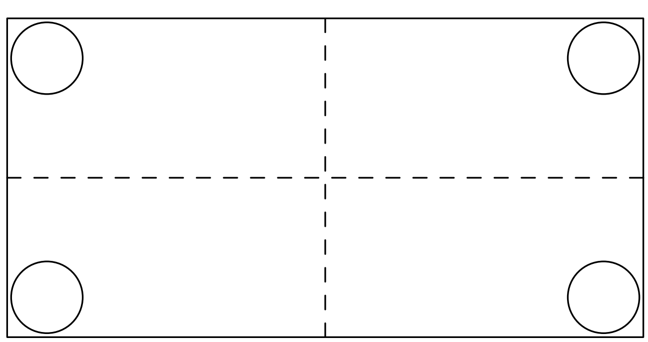
A



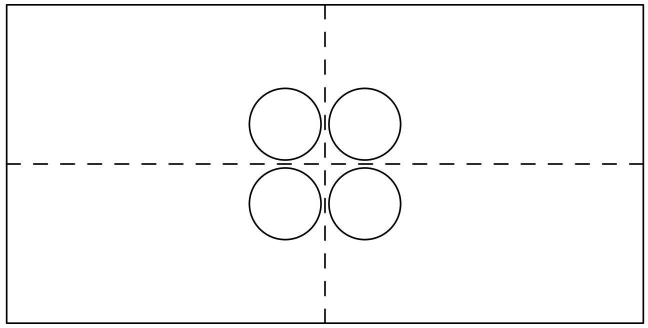
B



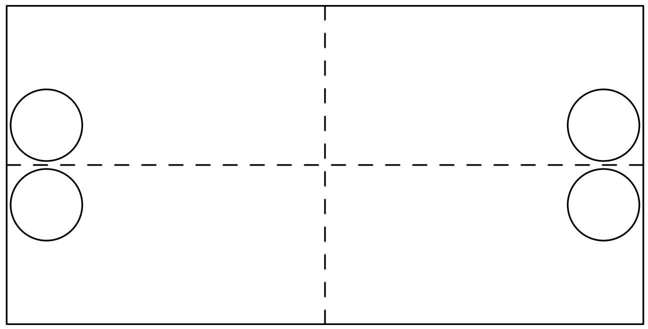
C



D



E



Solution(s):

When we undo the fold, the rectangle with the punched hole in the upper left is in the bottom right. The only such answer choice is **B**.

Thus, the correct answer is **B**.

19. Tamika selects two different numbers at random from the set $\{8, 9, 10\}$ and adds them. Carlos takes two different numbers at random from the set $\{3, 5, 6\}$ and multiplies them. What is the probability that Tamika's result is greater than Carlos' result?

A $\frac{4}{9}$

B $\frac{5}{9}$

C $\frac{1}{2}$

D $\frac{1}{3}$

E $\frac{2}{3}$

Solution(s):

Tamika, with equal probability, can get one of $17, 18, 19$.

Carlos, with equal probability, can get one of $15, 18, 30$.

There is a $\frac{1}{3}$ probability that Carlos gets 15, which would always have Tamika having a higher number.

There is a $\frac{1}{3}$ probability that Carlos gets 18, which would have a $\dfrac{1}{3}$ probability that Tamika gets a higer number of 19.

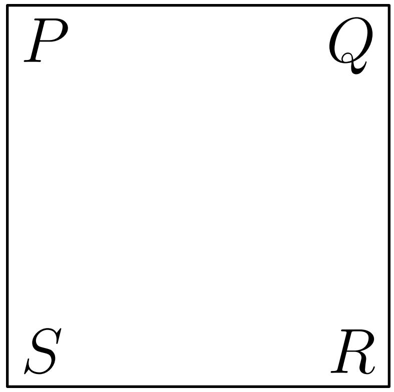
There is a $\frac{1}{3}$ probability that Carlos gets 30, which would always have Tamika never a higher number.

Therefore, the total probability is:

$$\frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} = \frac{4}{9}.$$

Thus, the correct answer is **A**.

20. Let $PQRS$ be a square piece of paper. P is folded onto R and then Q is folded onto S. The area of the resulting figure is 9 square inches. Find the perimeter of square $PQRS$.



A 9

B 16

C 18

D 24

E 36

Solution(s):

Let the side length be s. Then, folding P to R would give an isoceles right triangle with area $\frac{1}{2}s^2$. Here, Q and S are across from each other, so when it is folded, the area is $\frac{1}{4}s^2 = 9$, making $s = 6$.

As such, the area is $4 \cdot 6 = 24$.

Thus, the correct answer is **D**.

21. A $4 \times 4 \times 4$ cubical box contains 64 identical small cubes that exactly fill the box. How many of these small cubes touch a side or the bottom of the box?

A 48

B 52

C 60

D 64

E 80

Solution(s):

The entire $4 \times 4 \times 1$ bottom with a volume of 16 is counted. Then, there are 48 cubes left. There are 3 layers, with each of being a 4×4 square. Only the 2×2 interior doesn't touch the outside, so each layer has

$$4^2 - 2^2 = 12$$

cubes. This makes the total number of cubes equal to

$$12 \cdot 3 + 16 = 52.$$

Thus, the correct answer is **B**.

22. Terri produces a sequence of positive integers by following three rules. She starts with a positive integer, then applies the appropriate rule to the result, and continues in this fashion.

Rule 1: If the integer is less than 10, multiply it by 9.

Rule 2: If the integer is even and greater than 9, divide it by 2.

Rule 3: If the integer is odd and greater than 9, subtract 5 from it.

For example, consider the sample sequence: $23, 18, 9, 81, 76, \ldots$.

Find the 98^{th} term of the sequence that begins with:

$$98, 49, \ldots$$

A 6

B 11

C 22

D 27

E 54

Solution(s):

The sequence starts with the following:

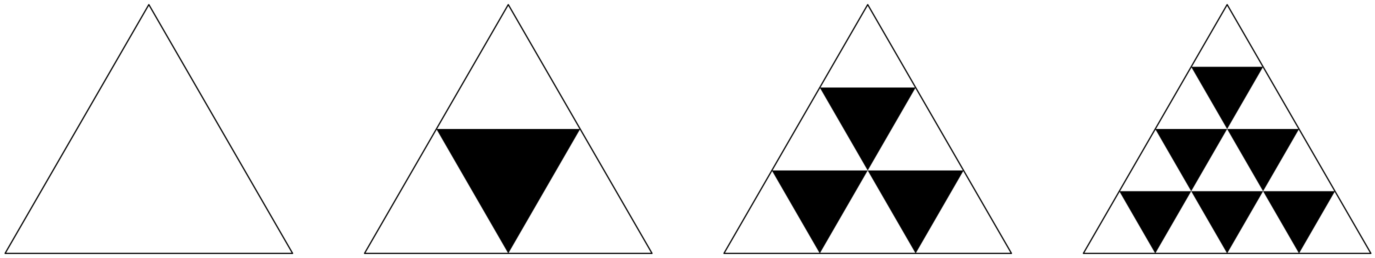
$$98, 49, 44, 22, 11,$$

$$6, 54, 27, 22, \cdots$$

Thus, after the first four, it has a cycle of length 5. This makes the 98^{th} term equal to the 8^{th} term of the sequence, which is 27.

Thus, the correct answer is **D**.

23. If the pattern in the diagram continues, what fraction of eighth triangle would be shaded?



A $\frac{3}{8}$

B $\frac{5}{27}$

C $\frac{7}{16}$

D $\frac{9}{16}$

E $\frac{11}{45}$

Solution(s):

The total number of triangles in the n^{th} triangle is n^2.

The total number of shaded triangles in the n^{th} triangle is the $n-1^{th}$ triangular number, which is:

$$\frac{n^2-n}{2}.$$

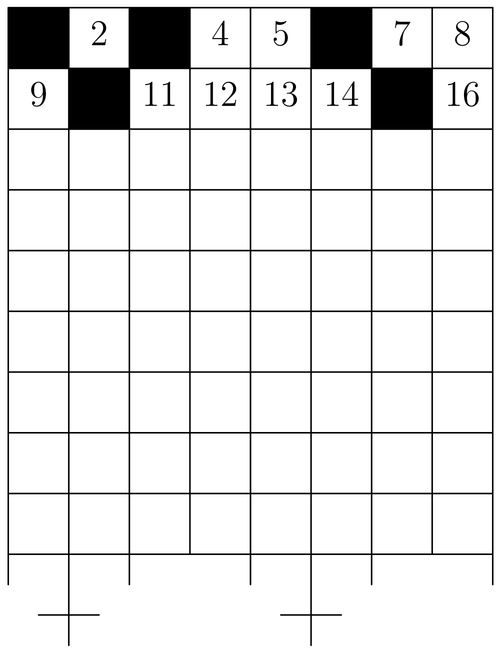
This makes the ratio equal to:

$$\frac{n-1}{2n}=\frac{7}{16}.$$

Thus, the correct answer is **C**.

24. A rectangular board of 8 columns has squares numbered beginning in the upper left corner and moving left to right so row one is numbered 1 through 8, row two is 9 through 16, and so on. A student shades square 1, then skips one square and shades square 3, skips two squares and shades square 6, skips 3 squares and shades square 10, and continues in this way until there is at least one shaded square in each column.

What is the number of the shaded square that first achieves this result?



- A 36
- B 64
- C 78
- D 91
- E 120

Solution(s):

The only shaded squares are the triangular numbers (the n^{th} triangular number is the sum of the first n numbers). As such, we have to find the first triangular number such that every possible value $\mod 8$ is accounted for.

Note that the formula for triangular numbers is

$$\frac{n^2+n}{2}.$$

Thus, we must find a triangular number for each value modulo 8. For $0 \mod 8$, the first one is 120, meaning the answer is at least 120. 120 is the greatest answer choice available, so we know the answer must be 120. By inspection, we can also see that every other value modulo 8 comes before anyways, but that is omitted here.

Thus, the correct answer is **E**.

25. Three generous friends, each with some money, redistribute the money as followed: Amy gives enough money to Jan and Toy to double each amount has. Jan then gives enough to Amy and Toy to double their amounts. Finally, Toy gives enough to Amy and Jan to double their amounts. If Toy had $36 at the beginning and $36 at the end, what is the total amount that all three friends have?

A $108

B $180

C $216

D $252

E $288

Solution(s):

Since Toy doubles his money after the first two turns, he has $144 in the end. Then, he gives away 144-36=$108. Since this doubles Any and Jan's money, they had $108 before Toy gives them money. Thus, the total amount of money is $144+$108=$252.

Thus, the correct answer is **D**.

Problems: https://live.poshenloh.com/past-contests/amc8/1998