2002 AMC 8 Solutions

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1. A circle and two distinct lines are drawn on a sheet of paper. What is the largest possible number of points of intersection of these figures?

A 2

B 3

C 4

D 5

E 6

Solution(s):

A line and circle can only intersect each other at at most two points. Two lines can only intersect at one point.

Therefore, the maximum number of intersections is

$$2 \cdot 2 + 1 = 5.$$

Thus, **D** is the correct answer.

2. How many different combinations of \$5 bills and \$2 bills can be used to make a total of \$17? Order does not matter in this problem.

A 2

B 3

C 4

D 5

E 6

Solution(s):

We cannot use 4 or more dollar \$5 bills since that would go over the total.

We can use three \$5 bills and one \$2 bill. We cannot use two \$5 bills since we cannot make an odd amount of money with \$2 bills.

Finally, we can use one \$5 bill and six \$2 bills. As above, we cannot use zero \$5 bills. Therefore, there are 2 ways to get the total.

Thus, **A** is the correct answer.

3. What is the smallest possible average of four distinct positive even integers?

A 3

B 4

C 5

D 6

E 7

Solution(s):

To get the smallest possible average, we want to use the smallest 4 positive even integers.

This can be achieved as follows:

$$\frac{2+4+6+8}{4} = \frac{20}{4}$$
$$= 5.$$

Thus, **C** is the correct answer.

4. The year 2002 is a palindrome (a number that reads the same from left to right as it does from right to left). What is the product of the digits of the next year after 2002 that is a palindrome?

A 0

B 4

C 9

D 16

E 25

Solution(s):

We don't want to increase the thousands digit, so we can keep that as 2.

This means that we have to increase the tens and hundreds digits to 1, to yield the next palindrome of 2112. The product of its digits is 4.

Thus, **B** is the correct answer.

5. Carlos Montado was born on Saturday, November $9, 2002$. On what day of the week will Carlos be 706 days old?

A Monday

B Wednesday

C Friday

D Saturday

E Sunday

Solution(s):

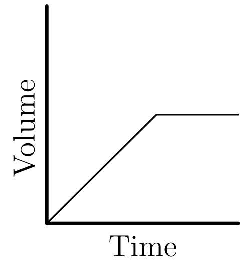
The days of the week cycle every 7 days. After 700 days, the day of the week will still be Saturday.

After 6 more days, the day of the week will be Friday.

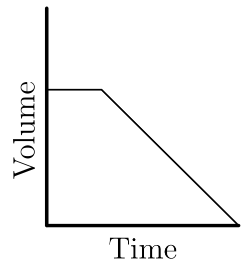
Thus, **C** is the correct answer.

6. A birdbath is designed to overflow so that it will be self-cleaning. Water flows in at the rate of 20 milliliters per minute and drains at the rate of 18 milliliters per minute. One of these graphs shows the volume of water in the birdbath during the filling time and continuing into the overflow time. Which one is it?

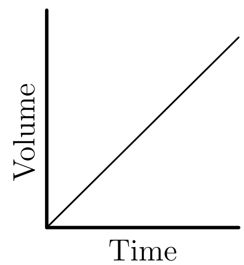
A



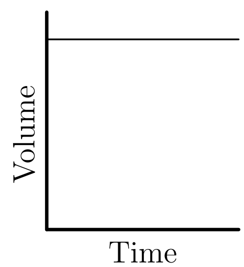
B



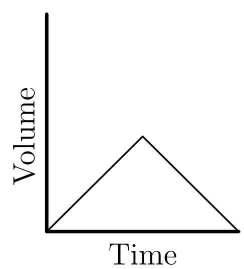
C



D



E



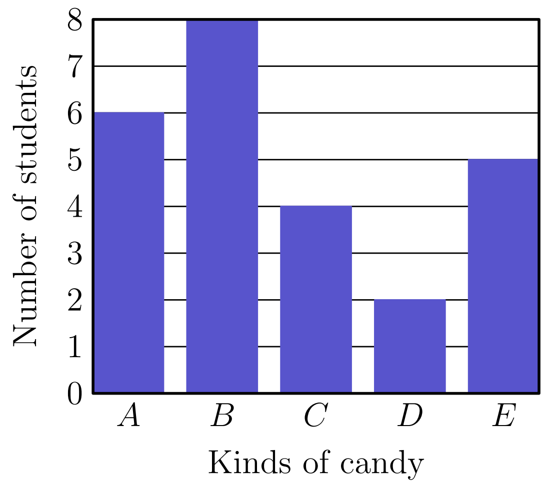
Solution(s):

The birdbath gains $20 - 18 = 2$ milliliters of water per minute.

The birdbath will continue to gain water until it is fill, when the volume will remain constant.

Thus, **A** is the correct answer.

7. The students in Mrs. Sawyer's class were asked to do a taste test of five kinds of candy. Each student chose one kind of candy. A bar graph of their preferences is shown. What percent of her class chose candy E?



A 5

B 12

C 15

D 16

E 20

Solution(s):

There are a total of

$$6+8+4+2+5=25$$

students in the class. The percent that chose E is

$$100 \cdot \frac{5}{25}=\frac{100}{5}=20\%.$$

Thus, **E** is the correct answer.

8. Juan organizes the stamps in his collection by country and by the decade in which they were issued. The prices he paid for them at a stamp shop were: Brazil and France, $6¢$ each, Peru $4¢$ each, and Spain $5¢$ each. (Brazil and Peru are South American countries and France and Spain are in Europe.)

Number of Stamps by Decade

Country	'50s	'60s	'70s	'80s
Brazil	4	7	12	8
France	8	4	12	15
Peru	6	4	6	10
Spain	3	9	13	9

How many of his European stamps were issued in the '80s?

A 9

B 15

C 18

D 24

E 42

Solution(s):

Note that France and Spain are the European countries. The number of '80s stamps from these countries respectively is 15 and 9 for a total of

$$15 + 9 = 24$$

stamps.

Thus, **D** is the correct answer.

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Peru	6	4	6	10
Spain	3	9	13	9

His South American stamps issued before the '70s cost him

A $0.40

B $1.06

C $1.80

D $2.38

E $2.64

Solution(s):

Note that Brazil and Peru are the South American countries.

Brazil's '50 s and '60 s total 11 stamps with a cost of $11 \cdot 6 = 66¢$.

Peru's '50 s and '60 s total 10 stamps with a cost of $10 \cdot 4 = 40¢$.

Therefore, the total cost is

$$66¢ + 40¢ = 106¢$$

= $1.06.

Thus, **B** is the correct answer.

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The average price of his '70 s stamps is closes to

A $3.5¢$

B $4¢$

C $4.5¢$

D $5¢$

E $5.5¢$

Solution(s):

The total price of all the '70 s is

$$12 \cdot 6 + 12 \cdot 6 + 6 \cdot 4 + 13 \cdot 5$$

$$= 72 + 72 + 24 + 65$$

$$= 233¢.$$

The total number of stamps is

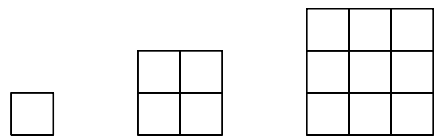
$$12 + 12 + 6 + 13 = 43.$$

Therefore, the average is

$$233 \div 43 \approx 5.4¢.$$

Thus, **E** is the correct answer.

11. A sequence of squares is made of identical square tiles. The edge of each square is one tile length longer than the edge of the previous square. The first three squares are shown. How many more tiles does the seventh square require than the sixth?



A 11

B 12

C 13

D 14

E 15

Solution(s):

Note that the number of tiles the n th square requires is n^2 since each side has n tiles.

Therefore, the 6 th square will need $6^2 = 36$ tiles and 7 th will need $7^2 = 49$. The difference is $49 - 36 = 13$.

Thus, **C** is the correct answer.

12. A board game spinner is divided into three regions labeled A, B and C. The probability of the arrow stopping on region A is $\frac{1}{3}$ and on region B is $\frac{1}{2}$. The probability of the arrow stopping on region C is

A $\dfrac{1}{12}$

B $\dfrac{1}{6}$

C $\dfrac{1}{5}$

D $\dfrac{1}{3}$

E $\dfrac{2}{5}$

Solution(s):

The total probability is 1. We need to subtract the probability of the spinner landing on B and A to get C, which is

$$1-\frac{1}{3}-\frac{1}{2}=\frac{1}{6}.$$

Thus, **B** is the correct answer.

13. For his birthday, Bert gets a box that holds 125 jellybeans when filled to capacity. A few weeks later, Carrie gets a larger box full of jellybeans. Her box is twice as high, twice as wide and twice as long as Bert's. Approximately, how many jellybeans did Carrie get?

- A 250
- B 500
- C 625
- D 750
- E 1000

Solution(s):

The larger box will have approximately

$$125 \cdot 2 \cdot 2 \cdot 2 = 1000$$

jellybeans.

Thus, **E** is the correct answer.

14. A merchant offers a large group of items at 30% off. Later, the merchant takes 20% off these sale prices. The total discount is

A 35%

B 44%

C 50%

D 56%

E 60%

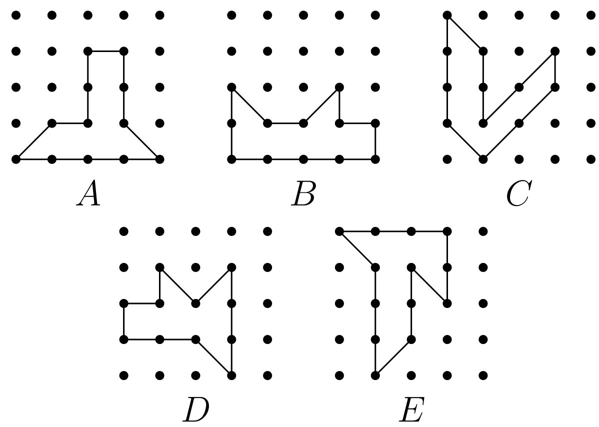
Solution(s):

let x be the original price of the items. 30% is $.7x$. Another 20% off is $.8 \cdot .7x = .56x$.

This means that the price is 44% less than the original price.

Thus, **B** is the correct answer.

15. Which of the following polygons has the largest area?



A A

B B

C C

D D

E E

Solution(s):

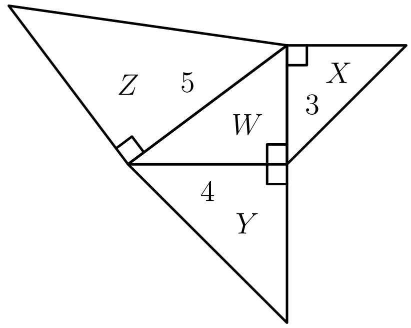
The number of boxes enclosed by each polygon can be obtained by dividing the polygon into unit squares and right triangles with sidelength 1 and adding up their values.

The unit squares count as 1 and the triangles count as $.5$.

A has a total area of 5, B has 5, C has 5, D has 4.5, and E has 5.5.

Thus, **E** is the correct answer.

16. Right isosceles triangles are constructed on the sides of a $3-4-5$ right triangle, as shown. A capital letter represents the area of each triangle. Which one of the following is true?



A $X+Z=W+Y$

B $W+X=Z$

C $3X+4Y=5Z$

D $X+W=\frac{1}{2}(Y+Z)$

E $X+Y=Z$

Solution(s):

We can find the area of all the triangles since we know both legs.

$$W=\frac{3\cdot 4}{2}=6$$
$$X=\frac{3\cdot 3}{2}=4.5$$
$$Y=\frac{4\cdot 4}{2}=8$$
$$Z=\frac{5\cdot 5}{2}=12.5$$

Plugging these values into the answer choices, we see that $X+Y=Z$ is the only that is true.

Thus, **E** is the correct answer.

17. In a mathematics contest with ten problems, a student gains 5 points for a correct answer and loses 2 points for an incorrect answer. If Olivia answered every problem and her score was 29, how many correct answers did she have?

A 5

B 6

C 7

D 8

E 9

Solution(s):

Let x be the number of correct answers. Then she answered $10 - x$ questions incorrectly.

This gives her a total score of

$$5x - 2(10 - x) = 7x - 20.$$

We know that this equals 29, and solving yields

$$7x - 20 = 29$$

$$x = 7.$$

Thus, **C** is the correct answer.

18. Gage skated 1 hr 15 min each day for 5 days and 1 hr 30 min each day for 3 days. How long would he have to skate the ninth day in order to average 85 minutes of skating each day for the entire time?

A 1 hr

B 1 hr 10 min

C 1 hr 20 min

D 1 hr 40 min

E 2 hr

Solution(s):

Gage has skated a total of

$$5 \cdot 75 + 3 \cdot 90 = 645 \text{ min.}$$

For an average of 85 minutes over 9 days, Gage must have skated a total of

$$85 \cdot 9 = 765 \text{ min.}$$

This means that Gage must skate

$$765 - 645 = 120 \text{ min.}$$

on the last day. Note that 120 minutes is the same as 2 hours.

Thus, **E** is the correct answer.

19. How many whole numbers between 99 and 999 contain exactly one 0?

- A 72
- B 90
- C 144
- D 162
- E 180

Solution(s):

Note that the 0 digit can either be the tens or the units digit. This gives us 2 options for this.

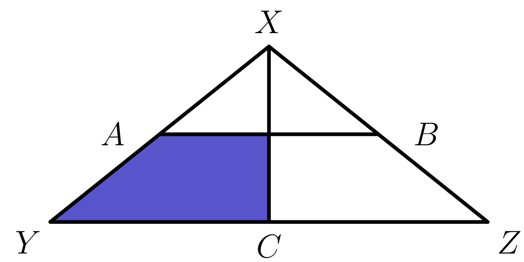
There are 9 options for each of the other digits for a total of

$$2 \cdot 9 \cdot 9 = 162$$

numbers.

Thus, **D** is the correct answer.

20. The area of triangle XYZ is 8 square inches. Points A and B are midpoints of congruent segments $\overline{XY}$ and $\overline{XZ}$. Altitude $\overline{XC}$ bisects $\overline{YZ}$. The area (in square inches) of the shaded region is



A $1\frac{1}{2}$

B 2

C $2\frac{1}{2}$

D 3

E $3\frac{1}{2}$

Solution(s):

Note that the area of $\triangle XYC$ is $8 \div 2 = 4$ since $\overline{XC}$ splits $\triangle XYZ$ into two congruent triangles.

We also know that the unshaded region is $\frac{1}{4}$ the area of the whole triangle, since all the sides are $\frac{1}{2}$ the length of the larger triangle.

This means that the shaded region is $\frac{3}{4}$ the area of the whole triangle, which is $4 \cdot \frac{3}{4} = 3$.

Thus, **D** is the correct answer.

21. Harold tosses a coin four times. The probability that he gets at least as many heads as tails is

A $\frac{5}{16}$

B $\frac{3}{8}$

C $\frac{1}{2}$

D $\frac{5}{8}$

E $\frac{11}{16}$

Solution(s):

The probability that there are at least as many head as tails is the same as the probability that there are at least as many tails as heads.

The only overlap between these two scenarios is when the number of heads and the number of tails is equal.

Let p be the desired probability. Then from our analysis we get that

$$p + p - q = 1,$$

where q is the probability of getting the same number of heads and tails.

To find q, there are $\binom{4}{2} = 6$ ways to choose which coins are heads, and there are a total of $2^4 = 16$ possibilities.

Therefore,

$$q = \frac{6}{16} = \frac{3}{8},$$

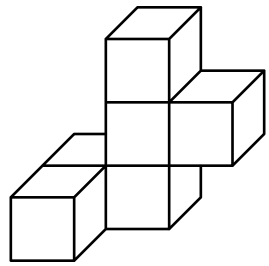
and

$$2p - \frac{3}{8} = 1$$

$$p = \frac{11}{16}.$$

Thus, **E** is the correct answer.

22. Six cubes, each an inch on an edge, are fastened together, as shown. Find the total surface area in square inches. Include the top, bottom, and sides.



A 18

B 24

C 26

D 30

E 36

Solution(s):

We can count the number of unexposed sides to find how many sides contribute to the surface area.

Three cubes have 1 side unexposed, two cubes have 2 sides unexposed, and one cube has 3 sides unexposed.

This gives us a total of

$$3 \cdot 1 + 2 \cdot 2 + 1 \cdot 3 = 10$$

unexposed sides, which gives us

$$6 \cdot 6 - 10 = 36 - 10$$

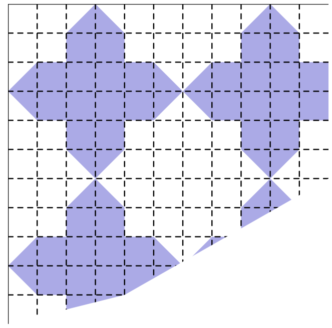
$$= 26$$

exposed sides.

Each exposed side contributes $1^2 = 1$ to the surface area, for a total surface area of $1 \cdot 26 = 26$.

Thus, **C** is the correct answer.

23. A corner of a tiled floor is shown. If the entire floor is tiled in this way and each of the four corners looks like this one, then what fraction of the tiled floor is made of darker tiles?



A $\frac{1}{3}$

B $\frac{4}{9}$

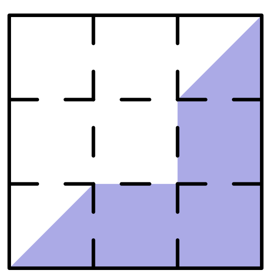
C $\frac{1}{2}$

D $\frac{5}{9}$

E $\frac{5}{8}$

Solution(s):

Notice that there are repeating 3×3 regions with the same pattern (they might be rotated differently).



In this region, there are three unit squares, and two triangles that combine to form another unit square.

This makes the area of the darker region 4 and the whole region 9. The desired fraction is then $\frac{4}{9}$.

Thus, **B** is the correct answer.

24. Miki has a dozen oranges of the same size and a dozen pears of the same size. Miki uses her juicer to extract 8 ounces of pear juice from 3 pears and 8 ounces of orange juice from 2 oranges. She makes a pear-orange juice blend from an equal number of pears and oranges. What percent of the blend is pear juice?

- A 30
- **B** 40
- C 50
- D 60
- E 70

Solution(s):

We can set up a proportion to find the amount of juice Miki can extract from the fruits:

$$\frac{p}{12} = \frac{8}{3}$$

for the pears and

$$\frac{o}{12} = \frac{8}{2}$$

for the oranges. Solving yields

$$p = 32 \text{ and } o = 48.$$

The percent of the whole that is pear juice is

$$\begin{aligned} 100 \cdot \frac{32}{32+48} &= 100 \cdot \frac{2}{5} \\ &= 40\%. \end{aligned}$$

Thus, **B** is the correct answer.

25. Loki, Moe, Nick and Ott are good friends. Ott had no money, but the others did. Moe gave Ott one-fifth of his money, Loki gave Ott one-fourth of his money and Nick gave Ott one-third of his money. Each gave Ott the same amount of money. What fractional part of the group's money does Ott now have?

A $\frac{1}{10}$

B $\frac{1}{4}$

C $\frac{1}{3}$

D $\frac{2}{5}$

E $\frac{1}{2}$

Solution(s):

WLOG, assume that everyone gave Ott \$1. This means that Moe had \$5, Loki had \$4, and Nick had \$3 originally.

Ott now has \$3, and the total amount of money is \$5 + \$4 + \$3 = \$12.

This means that Ott has

$$\frac{3}{12} = \frac{1}{4}$$

of the group's money.

Thus, **B** is the correct answer.

Problems: https://live.poshenloh.com/past-contests/amc8/2002