2003 AMC 8 Solutions

Typeset by: LIVE, by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc8/2003/solutions

1. Jamie counted the number of edges of a cube, Jimmy counted the numbers of corners, and Judy counted the number of faces. They then added the three numbers. What was the resulting sum?

A 12

B 16

C 20

D 22

E 26

Solution(s):

A cube has 12 edges, 8 corners, and 6 faces. Adding these together yields

$$12 + 8 + 6 = 26.$$

Thus, **E** is the correct answer.

2. Which of the following numbers has the smallest prime factor?

A 55

B 57

C 58

D 59

E 61

Solution(s):

Note that the smallest prime number is 2. This means that any even number would be our answer.

Thus, **C** is the correct answer.

3. A burger at Ricky C's weighs 120 grams, of which 30 grams are filler. What percent of the burger is not filler?

A 60%

B 65%

C 70%

D 75%

E 90%

Solution(s):

We get that $120 - 30 = 90$ grams are not filler. The percentage is therefore

$$100 \cdot \frac{90}{120} = 100 \cdot \frac{3}{4} = 75\%.$$

Thus, **D** is the correct answer.

4. A group of children riding on bicycles and tricycles rode past Billy Bob's house. Billy Bob counted 7 children and 19 wheels. How many tricycles were there?

A 2

B 4

C 5

D 6

E 7

Solution(s):

Let b be the number of bicycles and t be the number of tricycles. Then we can set up the following system of equations:

$$b + t = 7,$$
$$2b + 3t = 19.$$

Multiplying the first equation by 2 and subtracting from the second equation, we get $t = 5$.

Thus, **C** is the correct answer.

5. If 20% of a number is 12, what is 30% of the same number?

A 15

B 18

C 20

D 24

E 30

Solution(s):

Let x the be the number. Then

$$.2x = 12$$

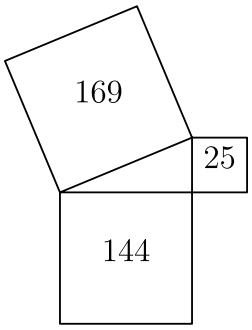
$$x = 60.$$

From this we get that

$$.3 \cdot 60 = 18.$$

Thus, **B** is the correct answer.

6. Given the areas of the three squares in the figure, what is the area of the interior triangle?



A 13

B 30

C 60

D 300

E 1800

Solution(s):

We get that the side lengths of the squares are

$$\sqrt{169} = 13$$

$$\sqrt{144} = 12,$$

$$\sqrt{25} = 5.$$

respectively. Note that these lengths form a Pythagorean triple.

Therefore, the interior triangle is right. Its area is

$$\frac{1}{2} \cdot 5 \cdot 12 = 30.$$

Thus, **B** is the correct answer.

7. Blake and Jenny each took four 100-point tests. Blake averaged 78 on the four tests. Jenny scored 10 points higher than Blake on the first test, 10 points lower than him on the second test, and 20 points higher on both the third and fourth tests. What is the difference between Jenny's average and Blake's average on these four tests?

A 10

B 15

C 20

D 25

E 40

Solution(s):

The total point difference between the two is

$$10 - 10 + 20 \cdot 2 = 40.$$

The average of this difference is $40 \div 4 = 10$.

Thus, **A** is the correct answer.

8. Who gets the fewest cookies from one batch of cookie dough?

A Art

B Roger

C Paul

D Trisha

E There is a tie for fewest.

Solution(s):

Note that the person who has the largest cookie would make the fewest cookies.

Art's cookie has an area of

$$\frac{1}{2}(3+5)\cdot 3 = 12 \text{ in}^2$$

Roger's is $2 \cdot 4 = 8 \text{ in}^2$, Paul's is $2 \cdot 3 = 6 \text{ in}^2$, and Trisha's is

$$\frac{1}{2} \cdot 3 \cdot 4 = 6 \text{ in}^2$$

Thus, **A** is the correct answer.

9. Art's cookies sell for $60¢$ each. To earn the same amount from a single batch, how much should one of Roger's cookies cost?

A $18¢$

B $25¢$

C $40¢$

D $75¢$

E $90¢$

Solution(s):

From Problem 8, we know that Art's cookie has an area of $12\ \text{in}^2$ Since there are 12 cookies in a batch, each batch has $12 \cdot 12 = 144\ \text{in}^2$ of dough.

Since Roger's cookie has an area of $8\ \text{in}^2$, Roger can make $144 \div 8 = 18$ cookies.

Art would make $12 \cdot 60 = 720¢$ from his cookies, so Roger would need to charge $720 \div 18 = 40¢$ per cookie.

Thus, **C** is the correct answer.

10. How many cookies will be in one batch of Trisha's cookies?

A 10

B 12

C 16

D 18

E 24

Solution(s):

From Problem 9, we know that a batch has $144\ \text{in}^2$ of cookie dough. We also know that Trisha's cookies have an area of $6\ \text{in}^2$

Therefore, Trisha can make $144 \div 6 = 24$ cookies per batch.

Thus, **E** is the correct answer.

11. Business is a little slow at Lou's Fine Shoes, so Lou decides to have a sale. On Friday, Lou increases all of Thursday's prices by 10%. Over the weekend, Lou advertises the sale: "Ten percent off the listed price. Sale starts Monday." How much does a pair of shoes cost on Monday that cost $40 on Thursday?

A $36

B $39.60

C $40

D $40.40

E $44

Solution(s):

On Friday, the shoes would cost 40 \cdot 1.1 = 44 $. On Monday, they would cost 44 \cdot 0.9 = 39.6 $.

Thus, **B** is the correct answer.

12. When a fair six-sided dice is tossed on a table top, the bottom face cannot be seen. What is the probability that the product of the faces that can be seen is divisible by 6?

A $\frac{1}{3}$

B $\frac{1}{2}$

C $\frac{2}{3}$

D $\frac{5}{6}$

E 1

Solution(s):

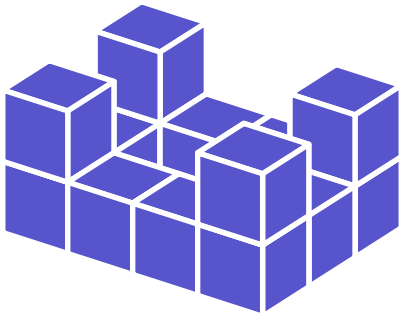
Note that if 6 is not the bottom case, then the product is automatically divisible by 6 since 6 is a factor.

If 6 is on the bottom, then 2 and 3 are seen, also ensuring that the product is divisible by 6.

Therefore, every possible scenario allows for the product to be divisible by 6.

Thus, **E** is the correct answer.

13. Fourteen white cubes are put together to form the figure on the right. The complete surface of the figure, including the bottom, is painted purple. The figure is then separated into individual cubes. How many of the individual cubes have exactly four purple faces?



A 4

B 6

C 8

D 10

E 12

Solution(s):

The 4 cubes on top have 5 exposed faces, so they don't work. The 4 corners in the bottom row have 3 exposed sides, so they don't work either.

Every other cube has 4 exposed sides, so those work. There are $14-4-4=6$ cubes that work.

Thus, **B** is the correct answer.

14. In this addition problem, each letter stands for a different digit.

$$\begin{array}{cccc} & T & W & O \\ + & T & W & O \\ \hline F & O & U & R \end{array}$$

If $T = 7$ and the letter O represents an even number, what is the only possible value for W?

A 0

B 1

C 2

D 3

E 4

Solution(s):

Since both T's are 7, we get that O is either 4 or 5. Since O is even, we get that $O = 4$.

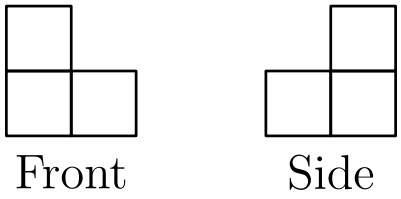
Then, we get that $R = 4 + 4 = 8$. We also know that $W + W$ doesn't carry over, since otherwise O would be 5.

Therefore, W is less than 5 and cannot be 4 or 1. If $W = 2$, then $U = 4$, which is also not allowed.

This makes $W = 3$.

Thus, **D** is the correct answer.

15. A figure is constructed from unit cubes. Each cube shares at least one face with another cube. What is the minimum number of cubes needed to build a figure with the front and side views shown?



A 3

B 4

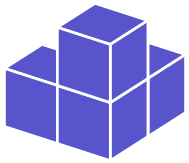
C 5

D 6

E 7

Solution(s):

We need at least 3 cubes to form the front view. Then we also need another cube to ensure that the side views are correct.



Thus, **B** is the correct answer.

16. Ali, Bonnie, Carlo and Dianna are going to drive together to a nearby theme park. The car they are using has four seats: one driver's seat, one front passenger seat and two back passenger seat. Bonnie and Carlo are the only ones who know how to drive the car. How many possible seating arrangements are there?

A 2

B 4

C 6

D 12

E 24

Solution(s):

There are 2 options for who sits in the driver's seat. There are 3 options for the other front seat, and 2 options for the first passenger seat.

The last person has to sit in the last seat, for a total of

$$2 \cdot 3 \cdot 2 = 12$$

possible seating arrangements.

Thus, **D** is the correct answer.

17. The six children listed below are from two families of three siblings each. Each child has blue or brown eyes and black or blond hair. Children from the same family have at least one of these characteristics in common. Which two children are Jim's siblings?

Child	Eye Color	Hair Color
Benjamin	Blue	Black
Jim	Brown	Blond
Nadeen	Brown	Black
Austin	Blue	Blond
Tevyn	Blue	Black
Sue	Blue	Blond

A Nadeen and Austin

B Benjamin and Sue

C Benjamin and Austin

D Nadeen and Tevyn

E Austin and Sue

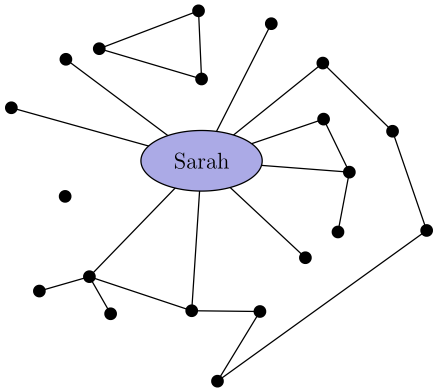
Solution(s):

Note that Nadeen, Austin, and Sue are the only individuals who share a characteristic with Jim. We need to find which of the 3 are completely different from the others.

Austin and Sue both have blue eyes, which makes Nadeen the odd one out. Therefore, Austin and Sue are Jim's siblings.

Thus, **E** is the correct answer.

18. Each of the twenty dots on the graph below represents one of Sarah's classmates. Classmates who are friends are connected with a line segment. For her birthday party, Sarah is inviting only the following: all of her friends and all of those classmates who are friends with at least one of her friends. How many classmates will not be invited to Sarah's party?



A 1

B 4

C 5

D 6

E 7

Solution(s):

Note that Sarah invites only the dots that are most 2 line segments away from Sarah.

There are 4 people who are completely disconnected from Sarah, and there are 2 people who are 3 lines away from Sarah.

Sarah will not invite any of these $4 + 2 = 6$ people.

Thus, **D** is the correct answer.

19. How many integers between 1000 and 2000 have all three of the numbers $15, 20,$ and 25 as factors?

A 1

B 2

C 3

D 4

E 5

Solution(s):

If a number x has these three numbers as factors, then their least common multiple must also divide x.

These numbers have the following prime factorizations:

$$15 = 3 \cdot 5,$$

$$20 = 2^2 \cdot 5,$$

$$25 = 5^2.$$

From these values, we get that the least common multiple is

$$2^2 \cdot 3 \cdot 5^2 = 300.$$

Therefore, the multiples of 300 between 1000 and 2000 are $1200, 1500,$ and 1800.

Thus, **C** is the correct answer.

20. What is the measure of the acute angle formed by the hands of the clock at $4:20$ a.m.?

A 0°

B 5°

C 8°

D 10°

E 12°

Solution(s):

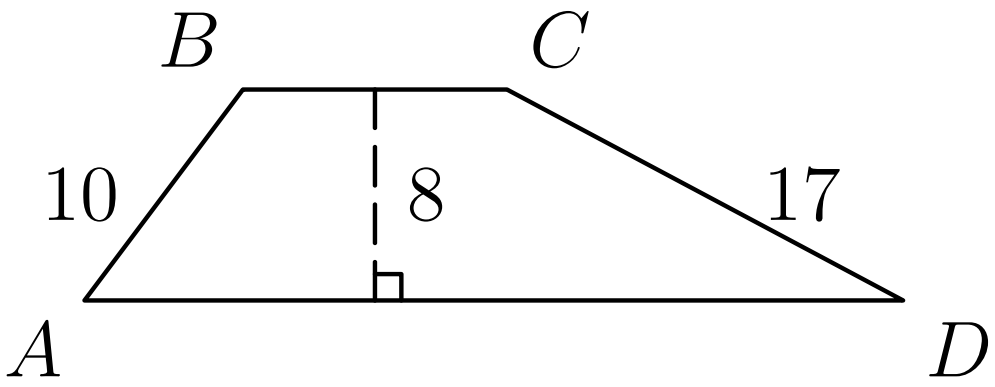
At $4:20$, the hour hand will be a $\frac{1}{3}$ of the way between 4 and 5.

Each hour represents $360 \div 12 = 30^\circ$. This means the hour hand will be $30 \div 3 = 10^\circ$ past 4.

Note that at 20 minutes, the minute hand will be at 4. This means the degree formed is 10°.

Thus, **D** is the correct answer.

21. The area of trapezoid $ABCD$ is 164 cm^2. The altitude is 8 cm, AB is 10 cm, and CD is 17 cm. What is BC, in centimeters?



A 9

B 10

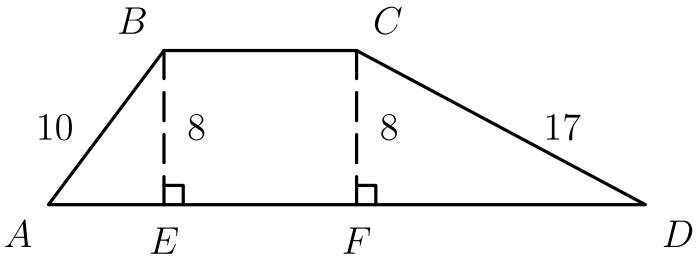
C 12

D 15

E 20

Solution(s):

Drop perpendiculars from B and C to $\overline{AD}$ and let them hit at E and F.



Then, using the Pythagorean theorem, we get that

$$AE = 6 \text{ and } FD = 15.$$

Then the area of $ABCD$ can be expressed as

$$[ABCD] =$$

$$[ABE] + [CDF] + [BCEF].$$

Note that:

$$[ABE] = \frac{1}{2} \cdot 6 \cdot 8 = 24$$

$$[CDF] = \frac{1}{2} \cdot 15 \cdot 8 = 60$$

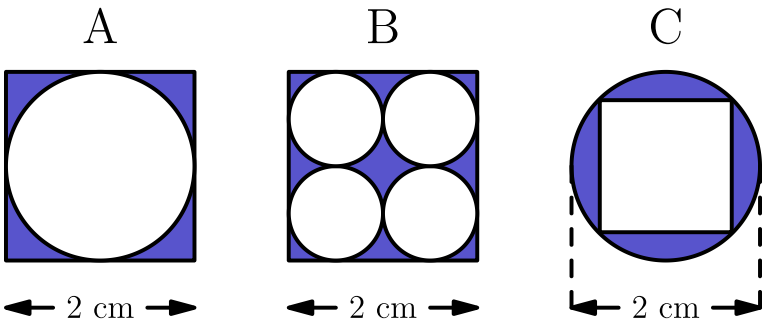
$$[BCEF] = 8 \cdot BC$$

Substituting, we get that

$$\begin{aligned} 164 &= 24 + 60 + 8BC \\ 80 &= 8BC \\ BC &= 10. \end{aligned}$$

Thus, **B** is the correct answer.

22. The following figures are composed of squares and circles. Which figure has a shaded region with largest area?



A A only

B B only

C C only

D both A and B

E all are equal

Solution(s):

The shaded area of A is

$$2^2 - 1^2\pi = 4 - \pi \approx .86 \text{ cm}^2$$

The shaded area of B is

$$2^2 - 4 \cdot \frac{1}{2}^2 \pi = 4 - \pi \approx .86 \text{ cm}^2$$

Note that the diagonal of the square in C is equal to the diameter of the circle.

Therefore, the side length is

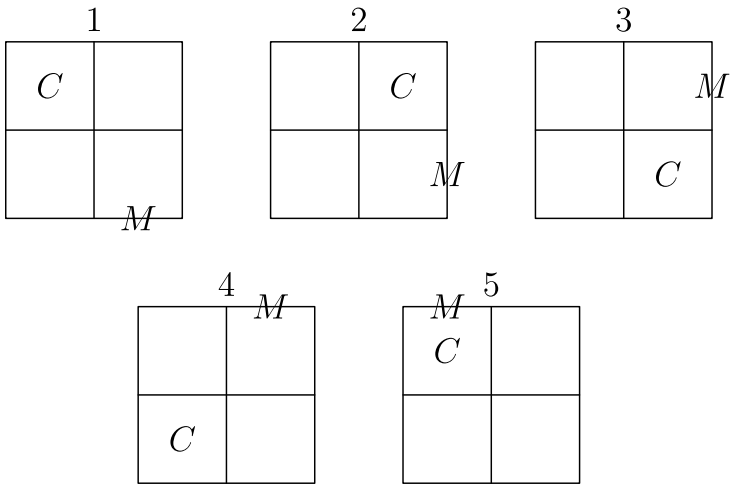
$$2 \div \sqrt{2} = \sqrt{2}.$$

This makes the area of the shaded region

$$1^2\pi - \sqrt{2}^2 = \pi - 2 \approx 1.14 \text{ cm}^2$$

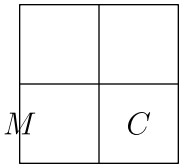
Thus, **C** is the correct answer.

23. In the pattern below, the cat moves clockwise through the four squares and the mouse moves counterclockwise through the eight exterior segments of the four squares.

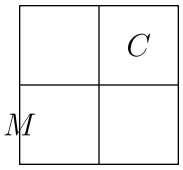


If the pattern is continued, where would the cat and mouse be after the 247th move?

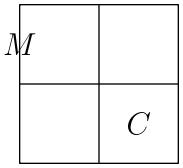
A



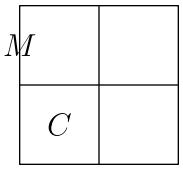
B



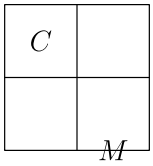
C



D



E



Solution(s):

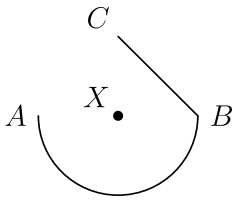
We can find the positions of the mouse and cat individually. Note that the mouse's position repeats every 8 moves and the cat's every 4 moves.

247 has a remainder of 3 when divided by 4, which means the cat is in the same position as after the 3rd move, which is the bottom right square.

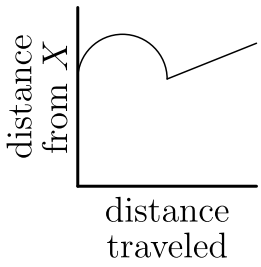
247 has a remainder of 7 when divided by 8, which means the cat is in the same position as after the 7th move, which is the bottom left segment.

Thus, **A** is the correct answer.

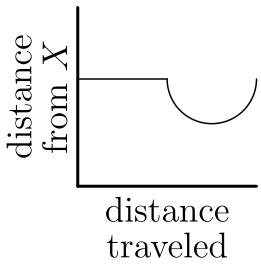
24. A ship travels from point A to point B along a semicircular path, centered at Island X. Then it travels along a straight path from B to C. Which of these graphs best shows the ship's distance from Island X as it moves along its course?



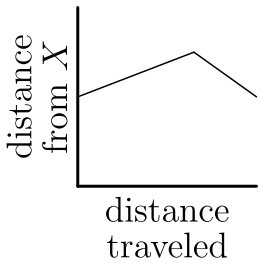
A



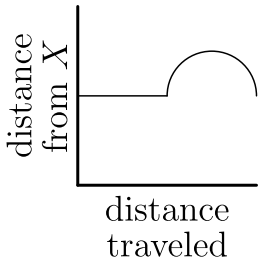
B



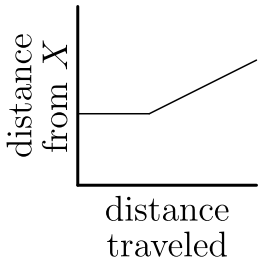
C



D



E



Solution(s):

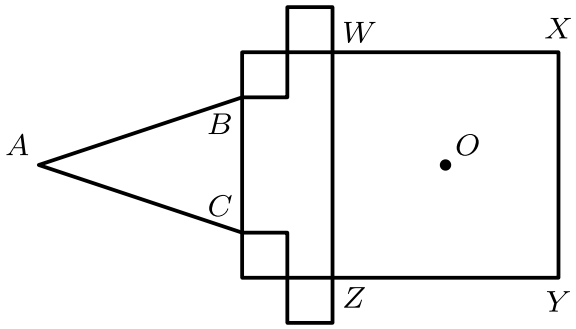
Note that any point along a circle is the same distance from its center. This means that travelling from A to B, the distance from X remains constant.

We also know that as we move from B to the midpoint of $\overline{BC}$, we get closer to X.

Similarly, as we move from the midpoint towards C, we get farther from X. This can be represented by a downwards facing semicircle.

Thus, **B** is the correct answer.

25. In the figure, the area of square $WXYZ$ is $25\ \text{cm}^2$. The four smaller squares have sides 1 cm long, either parallel to or coinciding with the sides of the large square. In $\triangle ABC$, $AB = AC$, and when $\triangle ABC$ is folded over side $\overline{BC}$, point A coincides with O, the center of square $WXYZ$. What is the area of $\triangle ABC$, in square centimeters?



A $\frac{15}{4}$

B $\frac{21}{4}$

C $\frac{27}{4}$

D $\frac{21}{2}$

E $\frac{27}{2}$

Solution(s):

We get that the side lengths of $WXYZ$ are 5 cm, since $\sqrt{25} = 5$.

We also know that the distance from $\overline{WZ}$ to $\overline{BC}$ is 2 since it is the sum of the side lengths of 2 unit squares.

Finally, the distance from A to $\overline{BC}$ is the same as the distance from $\overline{BC}$ to O, which is

$$2 + \frac{5}{2} = \frac{9}{2}\ \text{cm}$$

Now, we can find BC, which is

$$WZ - 2 = 5 - 2 = 3\ \text{cm}$$

Therefore, the area of $\triangle ABC$ is

$$\frac{1}{2} \cdot 3 \cdot \frac{9}{2} = \frac{27}{4} \text{ cm}^2$$

Thus, **C** is the correct answer.

Problems: https://live.poshenloh.com/past-contests/amc8/2003