2004 AMC 8 Solutions

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1. On a map, a 12-centimeter length represents 72 kilometers. How many kilometers does a 17-centimeter length represent?

A 6

B 102

C 204

D 864

E 1224

Solution(s):

Note that 1 cm represents $72 \div 12 = 6$ kilometers. This means that 17 cm represents $6 \cdot 17 = 102$ kilometers.

Thus, **B** is the correct answer.

2. How many different four-digit numbers can be formed be rearranging the four digits in 2004?

A 4

B 6

C 16

D 24

E 81

Solution(s):

Note that there are 2 non-zero digits that could be the thousands digit.

After choosing that, we need to arrange the other 3 digits. There are 3 spots for the other non-zero digit.

This gives us $2 \cdot 3 = 6$ possible numbers.

Thus, **B** is the correct answer.

3. Twelve friends met for dinner at Oscar's Overstuffed Oyster House, and each ordered one meal. The portions were so large, there was enough food for 18 people. If they shared, how many meals should they have ordered to have just enough food for the 12 of them?

- **A** 8
- **B** 9
- **C** 10
- **D** 15
- **E** 18

Solution(s):

Note that 12 meals feed 18 people. This means that one meal feeds $\frac{18}{12} = \frac{3}{2}$ people.

This means that they need $12 \div \frac{3}{2} = 8$ meals for 12 people.

Thus, **A** is the correct answer.

4. Ms. Hamilton's eighth-grade class wants to participate in the annual three-person-team basketball tournament. Lance, Sally, Joy, and Fred are chosen for the team. In how many ways can the three starters be chosen?

A 2

B 4

C 6

D 8

E 10

Solution(s):

If there are 3 starters, then one person must not be starting. Choosing the person who doesn't start determines the starters.

There are 4 choices for the person who doesn't start.

Thus, **B** is the correct answer.

5. Ms. Hamilton's eighth-grade class wants to participate in the annual three-person-team basketball tournament. The losing team of each game is eliminated from the tournament. If sixteen teams compete, how many games will be played to determine the winner?

- A 4
- B 7
- C 8
- D 15
- E 16

Solution(s):

Note that after every game, one team gets eliminated. For there to be one team remaining, 15 teams must have been eliminated.

This means that 15 games had to have been played.

Thus, **D** is the correct answer.

6. After Sally takes 20 shots, she has made 55% of her shots. After she takes 5 more shots, she raises her percentage to 56%. How many of the last 5 shots did she make?

- A 1
- B 2
- C 3
- D 4
- E 5

Solution(s):

Sally made $20 \times .55 = 11$ of her first 20 shots. Then we get that

$$\frac{11+x}{25} = .56,$$

which tells us that

$$11 + x = 14$$

and $x = 3$.

Thus, **C** is the correct answer.

7. An athlete's target heart rate, in beats per minute, is 80% of the theoretical maximum heart rate. The maximum heart rate is found by subtracting the athlete's age, in years, from 220. To the nearest whole number, what is the target heart rate of an athlete who is 26 years old?

A 134

B 155

C 176

D 194

E 243

Solution(s):

The maximum heart rate for this athlete would be $220 - 26 = 194$. Then the target heart rate would be $194 \times .8 \approx 155$.

Thus, **B** is the correct answer.

8. Find the number of two-digit positive integers whose digits total 7.

A 6

B 7

C 8

D 9

E 10

Solution(s):

Note that the tens digit can range from 1 to 7, and this digit determines the units digit.

Therefore, there are 7 numbers.

Thus, **B** is the correct answer.

9. The average of the five numbers in a list is 54. The average of the first two numbers is 48. What is the average of the last three numbers?

A 55

B 56

C 57

D 58

E 59

Solution(s):

The sum of all 5 numbers is $54 \cdot 5 = 270$. The sum of the first 2 numbers is $48 \cdot 2 = 96$.

The sum of the last 3 numbers is $270 - 96 = 174$. The average is therefore $174 \div 3 = 58$.

Thus, **D** is the correct answer.

10. Handy Aaron helped a neighbor $1\frac{1}{4}$ hours on Monday, 50 minutes on Tuesday, from $8:20$ to $10:45$ on Wednesday morning, and a half-hour on Friday. He is paid $ 3 per hour. How much did he earn for the week?

- A $ 8
- B $ 9
- C $ 10
- D $ 12
- E $ 15

Solution(s):

Aaron worked $\dfrac{50}{60}=\dfrac{5}{6}$ hours on Tuesday. He worked 2 hours and 25 minutes on Wednesday, which equals $2+\dfrac{25}{60}=\dfrac{29}{12}$ hours.

This means that he would have earned

$$3\left(\frac{5}{4}+\frac{5}{6}+\frac{29}{12}+\frac{1}{2}\right)$$

$$=3\cdot 5$$

= $ 15.

Thus, **E** is the correct answer.

11. The numbers $-2, 4, 6, 9$ and 12 are rearranged according to these rules:

1. The largest isn't first, but it is in one of the first three places.

2. The smallest isn't last, but it is in one of the last three places.

3. The median isn't first or last.

What is the average of the first and last numbers?

A 3.5

B 5

C 6.5

D 7.5

E 8

Solution(s):

Note that the largest, smallest, and median numbers cannot be the first or last number.

This means that the first and last numbers are 4 and 9 in some order. The average is

$$(4+9) \div 2$$

$$= 13 \div 2$$

$$= 6.5.$$

Thus, **C** is the correct answer.

12. Niki usually leaves her cell phone on. If her cell phone is on but she is not actually using it, the battery will last for 24 hours. If she is using it constantly, the battery will last for only 3 hours. Since the last recharge, her phone has been on 9 hours, and during that time she has used it for 60 minutes. If she doesn't use it any more but leaves the phone on, how many more hours will the battery last?

- A 7
- B 8
- C 11
- D 14
- E 15

Solution(s):

When not in use, her cell phone uses up $\frac{1}{24}$ of its battery per hour. When it is in use, it uses up $\frac{1}{3}$ of its battery per hour.

Niki's phone has been on for 9 hours, with 8 of those hours being idle and 1 hour being used to talk on the phone.

This means that the phone has used up $\frac{2}{3}$ of its battery. In order to drain the remaining $\frac{1}{3}$ of the battery, the phone can last for 8 more hours without being used.

Thus, **B** is the correct answer.

13. Amy, Bill and Celine are friends with different ages. Exactly one of the following statements is true.

I. Bill is the oldest.

II. Amy is not the oldest.

III. Celine is not the youngest.

Rank the friends from the oldest to youngest.

A Bill, Amy, Celine

B Amy, Bill, Celine

C Celine, Amy, Bill

D Celine, Bill, Amy

E Amy, Celine, Bill

Solution(s):

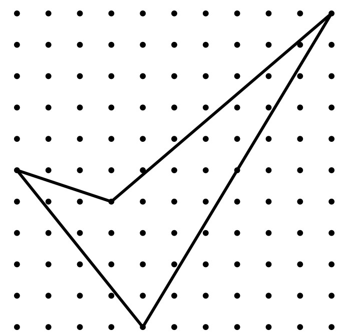
If Bill is the oldest, we get that statement I and II are true, which is not allowed. This tell us that statement I is false.

If Amy is not the oldest, then Celine is the oldest since Bill isn't. This makes statement III true, which is also not allowed.

Thus, statement III must the true one and I and II are false. Using this, we get the order to be Amy, Celine, Bill.

Thus, **E** is the correct answer.

14. What is the area enclosed by the geoboard quadrilateral below?



A 15

B $18\frac{1}{2}$

C $22\frac{1}{2}$

D 27

E 41

Solution(s):

We can use Pick's Theorem which tells us that the area of a polygon whose vertices are lattice points is given by

$$A = I + \frac{1}{2}B - 1,$$

where I is the number of interior lattice points and B is the number of boundary lattice points.

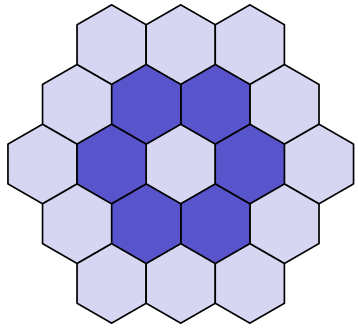
Alternatively, one could also decompose the quadrilateral into triangles, but it is also possible to prove Pick's Theorem by recursively decomposing arbitrary polygons into triangles with lattice point vertices, though that is left an exercise for the reader.

Counting, we get that $B = 5$ and $I = 21$. Plugging these in, we get

$$A = \frac{5}{2} + 21 - 1 = 22\frac{1}{2}.$$

Thus, **C** is the correct answer.

15. Thirteen dark and six bright hexagonal tiles were used to create the figure below. If a new figure is created by attaching a border of bright tiles with the same size and shape as the others, what will be the difference between the total number of bright tiles and the total number of dark tiles in the new figure?



A 5

B 7

C 11

D 12

E 18

Solution(s):

Note that the first ring around the middle tile has 6 tiles. The second ring has 12 tiles.

Following this pattern, we get that the third ring has 18 tiles. The total number of dark tiles is $1+12=13$ and bright tiles is $6+18=24$.

The difference is therefore $24-13=11$.

Thus, **C** is the correct answer.

16. Two 600 mL pitchers contain orange juice. One pitcher is $\frac{1}{3}$ full and the other pitcher is $\frac{2}{5}$ full. Water is added to fill each pitcher completely, then both pitchers are poured into one large container. What fraction of the mixture in the large container is orange juice?

A $\frac{1}{8}$

B $\frac{3}{16}$

C $\frac{11}{30}$

D $\frac{11}{19}$

E $\frac{11}{15}$

Solution(s):

The first pitcher contains $600 \cdot \dfrac{1}{3} = 200$ mL of orange juice. The second one has $600 \cdot \dfrac{2}{5} = 240$ mL.

The large container then has $200 + 240 = 440$ mL of orange juice. The total amount of mixture is $2 \cdot 600 = 1200$ mL.

Then the fraction of orange juice is $\dfrac{440}{1200} = \dfrac{11}{30}$.

Thus, **C** is the correct answer.

17. Three friends have a total of 6 identical pencils, and each one has at least one pencil. In how many ways can this happen?

- A 1
- B 3
- C 6
- D 10
- E 12

Solution(s):

Recall that stars and bars gives us the number of ways to put n objects into k bins, where each bin must have at least one object, by

$$\binom{n-1}{k-1}.$$

In our scenario, we must split 6 pencils amongst 3 people, where each person must have at least one pencil. Therefore, the number of ways is

$$\binom{6-1}{3-1} = \binom{5}{2} = 10.$$

Thus, **D** is the correct answer.

18. Five friends compete in a dart-throwing contest. Each one has two darts to throw at the same circular target, and each individual's score is the sum of the scores in the target regions that are hit. The scores for the target regions are the whole numbers 1 through 10. Each throw hits the target in a region with a different value. The scores are: Alice 16 points, Ben 4 points, Cindy 7 points, Dave 11 points, and Ellen 17 points. Who hits the region worth 6 points?

A Alice

B Ben

C Cindy

D Dave

E Ellen

Solution(s):

The only way to get Ben's score is with a 1 and 3 since he can't hit 2 twice.

Cindy can achieve her score with

$$1+6, 2+5, \text{ or } 3+4.$$

Ben already hit 1 and 3, so Cindy must have hit 2 and 5.

Similarly, Dave must have hit 4 and 7. Finally, since 7 is already used, Alice is forced to have hit 6 and 10 with Ellen hitting 8 and 9.

Thus, **A** is the correct answer.

19. A whole number larger than 2 leaves a remainder of 2 when divided by each of the numbers $3, 4, 5,$ and 6. The smallest such number lies between which two numbers?

A 40 and 49

B 60 and 79

C 100 and 129

D 210 and 249

E 320 and 369

Solution(s):

Let x be the number. Then $x - 2$ is divisible by $3, 4, 5,$ and 6.

The least common multiple of these numbers is 60, which makes x 62.

Thus, **B** is the correct answer.

20. Two-thirds of the people in a room are seated in three-fourths of the chairs. The rest of the people are standing. If there are 6 empty chairs, how many people are in the room?

A 12

B 18

C 24

D 27

E 36

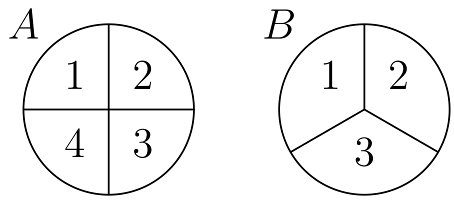
Solution(s):

Note that $\frac{1}{4}$ of the chairs are empty. Since this is equal to 6, there are $3 \cdot 4 = 18$ taken chairs.

If 18 is $\frac{2}{3}$ the number of people, then the total number of people is $18 \div \frac{2}{3} = 27$.

Thus, **D** is the correct answer.

21. Spinners A and B are spun. On each spinner, the arrow is equally likely to land on each number. What is the probability that the product of the two spinners' numbers is even?



A $\frac{1}{4}$

B $\frac{1}{3}$

C $\frac{1}{2}$

D $\frac{2}{3}$

E $\frac{3}{4}$

Solution(s):

For the product to be even, then at least one of the spinners must land on an even number.

We can use complementary counting and calculate the probability of both spinners landing on odds.

This happens with a probability of

$$\frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}.$$

Then the probability of landing on at least one even is

$$1 - \frac{1}{3} = \frac{2}{3}.$$

Thus, **D** is the correct answer.

22. At a party there are only single women and married men with their wives. The probability that a randomly selected woman is single is $\frac{2}{5}$. What fraction of the people in the room are married men?

A $\frac{1}{3}$

B $\frac{3}{8}$

C $\frac{2}{5}$

D $\frac{5}{12}$

E $\frac{3}{5}$

Solution(s):

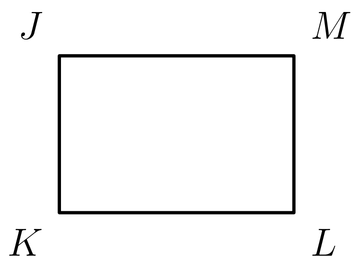
WLOG, let there be 5 women in the room. Then there are $5 \cdot \frac{2}{5} = 2$ single women.

This means that there are $5 - 2 = 3$ married women, which is also the number of married men.

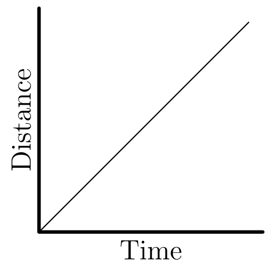
There are a total of $5 + 3 = 8$ people in the room. The fraction of married men is $\frac{3}{8}$.

Thus, **B** is the correct answer.

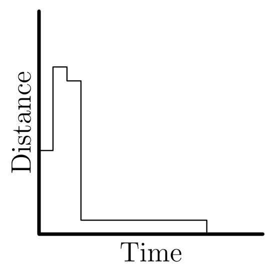
23. Tess runs counterclockwise around rectangular block $JKLM$. She lives at corner J. Which graph could represent her straight-line distance from home?



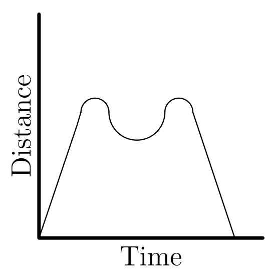
A



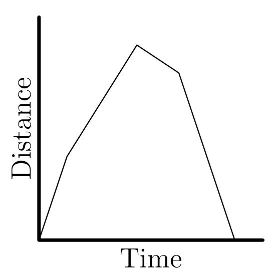
B



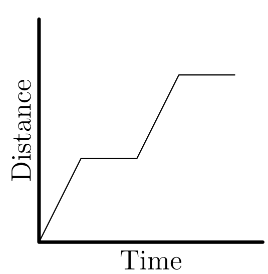
C



D



E



Solution(s):

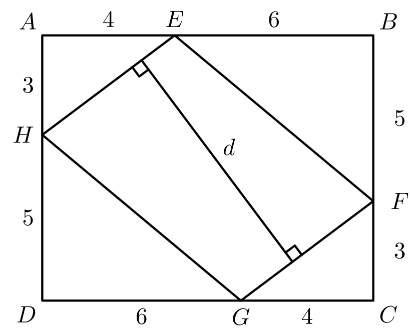
Tess cannot be represented by a constant horizontal line because this would imply that she is running in a circle centered at her starting point.

This means that **B** and **E** are incorrect as they involve straight lines.

A is incorrect because the maximum distance is at the end of Tess's journey, and **C** is wrong because it has 2 maximums.

Thus, **D** is the correct answer.

24. In the figure, $ABCD$ is a rectangle and $EFGH$ is a parallelogram. Using the measurements given in the figure, what is the length d of the segment that is perpendicular to $\overline{HE}$ and $\overline{FG}$?



A 6.8

B 7.1

C 7.6

D 7.8

E 8.1

Solution(s):

First, we can calculate the area of the parallelogram. This can be done be subtracting out the areas of the 4 triangles to get

$$10 \cdot 8 - 2 \cdot \frac{1}{2}(3 \cdot 4 + 6 \cdot 5)$$

$$= 80 - (12 + 30)$$

$$= 38.$$

We can also calculate the area of the parallelogram by multiplying the base by the altitude, d. Therefore

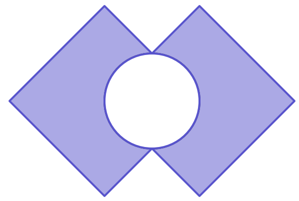
$$38 = 5d$$

$$d = \frac{38}{5} = 7.6$$

(we get the 5 from the Pythagorean theorem).

Thus, **C** is the correct answer.

25. Two 4×4 squares intersect at right angles, bisecting their intersecting sides, as shown. The circle's diameter is the segment between the two points of intersection. What is the area of the shaded region created by removing the circle from the squares?



A $16 - 4\pi$

B $16 - 2\pi$

C $28 - 4\pi$

D $28 - 2\pi$

E $32 - 2\pi$

Solution(s):

Note that the union of the two squares is a square with side length 2. This makes the area of the overlapping squares

$$4^2 + 4^2 - 2^2 = 28.$$

We need to remove the area of the circle, which has radius $\sqrt{2}$ (we get this since the diameter is the diagonal of a square with side length 2).

Therefore, the area of the shaded region is

$$28 - \pi\left(\sqrt{2}\right)^2 = 28 - 2\pi.$$

Thus, **D** is the correct answer.

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