2005 AMC 8 Solutions

1. Connie multiplies a number by 2 and gets 60 as her answer. However, she should have divided the number by 2 to get the correct answer. What is the correct answer?

   A | 7.5

   B | 15

   C | 30

   D | 120

   E | 240

   ## Solution(s):

   Since Connie multiplied by 2, her original number was $60 \div 2 = 30$. To get the correct answer, she must divide by 2 to get $30 \div 2 = 15$.

   Thus, **B** is the correct answer.

2. Karl bought five folders from Pay-A-Lot at a cost of $ 2.50 each. Pay-A-Lot had a 20%-off sale the following day. How much could Karl have saved on the purchase by waiting a day?

A	$ 1.00

B	$ 2.00

C	$ 2.50

D	$ 2.75

E	$ 5.00

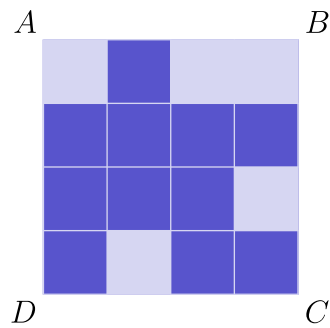## Solution(s):

Karl paid $5 \times \$ 2.5 = \$ 12.5$ for his folders. 20% off of this would have been $.2 \times \$ 12.5 = \$ 2.5$, which means Karl would have saved $ 2.5.

Thus, **C** is the correct answer.

3. What is the minimum number of small squares that must be colored dark so that a line of symmetry lies on the diagonal $\overline{BD}$ of square $ABCD$?

A   1

B   2

C   3

D   4

E   5

Solution(s):

Note that every dark square not along the diagonal must have a corresponding dark square in the other half.

Each of the 4 dark squares not on the diagonal do not have their corresponding square colored, so 4 additional squares must be colored dark.

Thus, **D** is the correct answer.

4. A square and a triangle have equal perimeters. The lengths of the three sides of the triangle are 6.1 cm, 8.2 cm and 9.7 cm. What is the area of the square in square centimeters?

A   24

B   25

C   36

D   48

E   64

Solution(s):

The perimeter of the triangle is

$$6.1 + 8.2 + 9.7 = 24 \text{ cm.}$$

This means that the side length of the square is

$$24 \div 4 = 6 \text{ cm.}$$

Therefore, the area of the square is

$$6^2 = 36 \text{ cm}^2.$$

Thus, **C** is the correct answer.

5. Soda is sold in packs of $6, 12$ and 24 cans. What is the minimum number of packs needed to buy exactly 90 cans of soda?

A   4

B   5

C   6

D   8

E   15

Solution(s):

To minimize the number of packs, we can use the largest packs first.

We can use three 24-packs to have

$$90 - 3 \cdot 24$$
$$= 90 - 72$$
$$= 18$$

cans left.

From this, we can see that we need one 12-pack and one 6-pack.

Therefore, we need $3 + 1 + 1 = 5$ packs.

Thus, **B** is the correct answer.

6. Suppose d is a digit. For how many values of d is

$$2.00d5 > 2.005?$$

A   0

B   4

C   5

D   6

E   10

Solution(s):

We can rearrange the inequality as follows.

$$2.00d5 > 2.005$$
$$.00d5 > .005$$
$$d.5 > 5$$
$$d > 4.5$$

From this, we can see that d must be greater than 5, yielding 5 values (5 through 9).

Thus, **C** is the correct answer.

7. Bill walks $\frac{1}{2}$ mile south, then $\frac{3}{4}$ mile east, and finally $\frac{1}{2}$ mile south. How many miles is he, in a direct line, from his starting point?
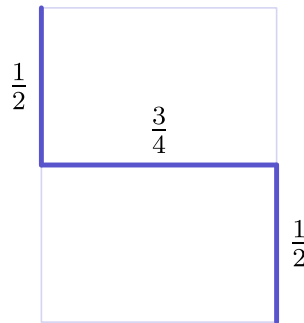
A   1

B $1\frac{1}{4}$

C $1\frac{1}{2}$

D $1\frac{3}{4}$

E   2

Solution(s):

Note that what we want is the length of the diagonal. We can use the Pythagorean theorem to find this.

$$\sqrt{\left(\frac{1}{2}+\frac{1}{2}\right)^2+\frac{3}{4}^2}$$

$$=\sqrt{1+\frac{9}{16}}=\sqrt{\frac{25}{16}}=\frac{5}{4}$$

Thus, **B** is the correct answer.

8. Suppose m and n are positive odd integers. Which of the following must also be an odd integer?

A $m + 3n$

B $3m - n$

C $3m^2 + 3n^2$

D $(nm + 3)^2$

E $3mn$

Solution(s):

Recall the four following rules:

• odd plus odd and even plus even is even

• even plus odd is odd

• even times anything is even

• odd times odd is odd

These rules can be easily verified by representing arbitrary odd numbers as $2m + 1$ and arbitrary even numbers as $2n$ respectively, for integers m, n.

With this in mind, let's examine each answer choice individually:

A:

Note that 3 is odd. This gives us

$$\text{odd} + \text{odd} \cdot \text{odd}.$$

From our above rules, we know that this is even.

B:

$$\text{odd} \cdot \text{odd} - \text{odd}.$$

Once again, this is even.

C:

$$\text{odd} \cdot \text{odd}^2 + \text{odd} \cdot \text{odd}^2.$$

This is also even.

D:

$$(odd \cdot odd + odd)^2.$$

Unfortunately, this is also even.

E:

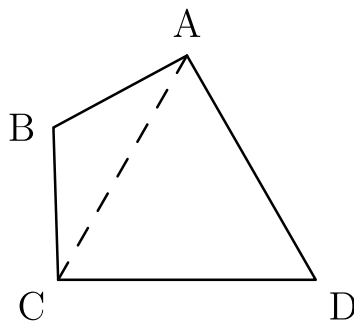$$odd \cdot odd \cdot odd.$$

This is odd.

Therefore, **E** is the only answer choice that is an odd integer.

Thus, **E** is the correct answer.

9. In quadrilateral $ABCD$, sides $\overline{AB}$ and $\overline{BC}$ both have length 10, sides $\overline{CD}$ and $\overline{DA}$ both have length 17, and the measure of angle ADC is $60°$. What is the length of diagonal $\overline{AC}$?

A	13.5

B	14

C	15.5

D	17

E	18.5

Solution(s):

Note that $\triangle ADC$ is isosceles. This means that $\angle DAC = \angle DCA$.

We get that

$$\angle DAC + \angle DCA + \angle ADC = 180°$$
$$\angle DAC + \angle DCA = 120°$$
$$\angle DAC = \angle DCA = 60°.$$

This shows that $\triangle ADC$ is equilateral. This gives us that

$$AC = CD = 17.$$

Thus, **D** is the correct answer.

10. Joe had walked half way from home to school when he realized he was late. He ran the rest of the way to school. He ran 3 times as fast as he walked. Joe took 6 minutes to walk half way to school. How many minutes did it take Joe to get from home to school?

A	7

B	7.3

C	7.7

D	8

E	8.3

Solution(s):

If Joe runs 3 times as fast as he walks, it would take him a third of the time to run the same distance versus walking.

This means that Joe will take $6 \div 3 = 2$ minutes to walk half way to school, for a total time of $6 + 2 = 8$ minutes.

Thus, **D** is the correct answer.

11. The sales tax rate in Bergville is 6%. During a sale at the Bergville Coat Closet, the price of a coat is discounted 20%. from its $ 90.00 price. Two clerks, Jack and Jill, calculate the bill independently. Jack rings up $ 90.00 and adds 6% sales tax, then subtracts 20% from this total. Jill rings up $ 90.00, subtracts 20% of the price, then adds 6% of the discounted price for sales tax. What is Jack's total minus Jill's total?

A   -$ 1.06

B   -$ 0.53

C $ 0

D $ 0.53

E $ 1.06

Solution(s):

Jack's total is

$$90.00 \times 1.06 \times .8$$

dollars. Jill's total is

$$90.00 \times .8 \times 1.06$$

dollars. Note that both these values are the same.

This means that the difference between the totals is $ 0.

Thus, **C** is the correct answer.

12. Big Al, the ape, ate 100 bananas from May 1 through May 5. Each day he ate six more bananas than on the previous day. How many bananas did Big Al eat on May 5?

A 20

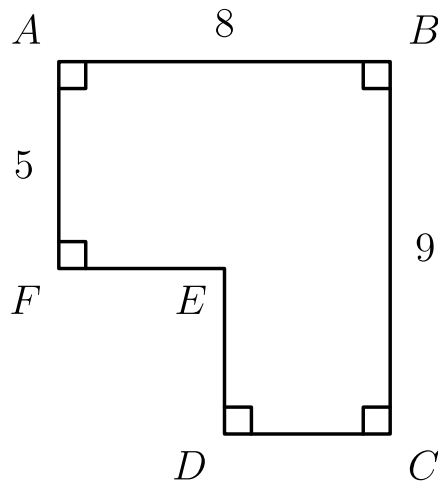B 22

C 30

D 32

E 34

Solution(s):

Let x be the number of bananas Big Al ate on May 3. Then Big Al ate the following amounts starting from May 1 :

$$x - 12, x - 6, x, x + 6, x + 12.$$

The sum of this is $5x$, which equals 100. This gives us that $x = 20$. Then on May 5, Big Al ate $20 + 12 = 32$ bananas.
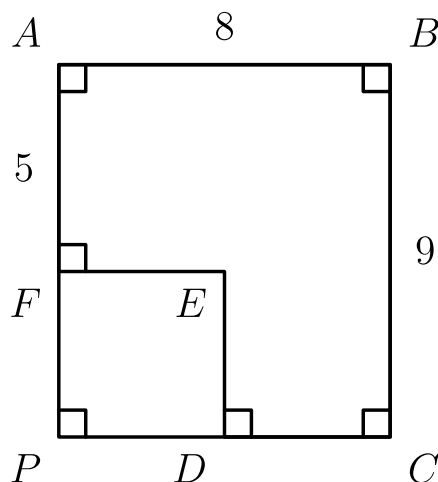
Thus, **D** is the correct answer.

13. The area of polygon $ABCDEF$ is 52 with $AB = 8$, $BC = 9$ and $FA = 5$. What is $DE + EF$?

A	7
B	8
C	9
D	10
E	11

Solution(s):

Extend $\overline{AF}$ and $\overline{CD}$ to meet at P. Then $ABCP$ is a rectangle with area $8 \cdot 9 = 72$.

Then the area of

$$FEDP = 72 - 52 = 20.$$

We know that
$$ED = BC - AF = 9 - 5 = 4.$$

Therefore,
$$FE = [FEDP] \div DE$$
$$= 20 \div 4 = 5.$$

The desired answer is therefore $4 + 5 = 9$.

Thus, **C** is the correct answer.

14. The Little Twelve Basketball Conference has two divisions, with six teams in each division. Each team plays each of the other teams in its own division twice and every team in the other division once. How many conference games are scheduled?

A 80

B 96

C 100

D 108

E 192

Solution(s):

Let us focus on one team. This team plays against 5 other teams twice in its own division, for a total of $2 \cdot 5 = 10$ games.

This team also plays 6 games with teams from the other division. Therefore, they play a total of $10 + 6 = 16$ games.

There are 12 teams total, so there are $12 \cdot 16 = 192$ games conducted. Each game, however, involves 2 teams, so we have to divide by 2.

This gives us the actual total number of games to be $192 \div 2 = 96$.

Thus, **B** is the correct answer.

15. How many different isosceles triangles have integer side lengths and perimeter 23?

A   2

B   4

C   6

D   9

E   11

Solution(s):

Let x be the length of the base of the triangle and y be the length of the legs.

We get that

$$2y + x = 23.$$

The triangle inequality tells us that $2a > b$. Plugging in the first equation, we get

$$2a > 23 - 2a$$

and therefore

$$a > 5.75.$$

This tells us that the minimum value of a is 6. We get the maximum value when we plug in $b = 1$ to get $a = 11$.

This gives us 6 values for a, yielding 6 different triangles.

Thus, **C** is the correct answer.

16. A five-legged Martian has a drawer full of socks, each of which is red, white or blue, and there are at least five socks of each color. The Martian pulls out one sock at a time without looking. How many socks must the Martian remove from the drawer to be certain there will be 5 socks of the same color?
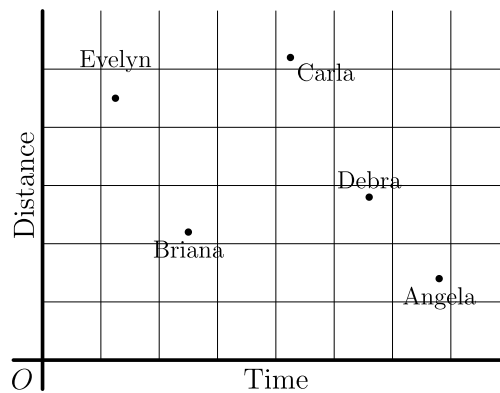
A | 6

B | 9

C | 12

D | 13

E | 15

Solution(s):

Note that the Martian can pull out 4 socks of each color without drawing 5 of the same color. The 13th sock, however, must be the 5th sock of some color.

Thus, **D** is the correct answer.

17. The results of a cross-country team's training run are graphed below. Which student has the greatest average speed?

A Angela

B Briana

C Carla

D Debra

E Evelyn

Solution(s):

Average speed is distance over time, which is given by the slope of the line through the point and the origin.

Evelyn has the steepest slope, telling us that she had the greatest average speed.

Thus, **E** is the correct answer.

18. How many three-digit numbers are divisible by 13?

A   7

B   67

C   69

D   76

E   77

Solution(s):

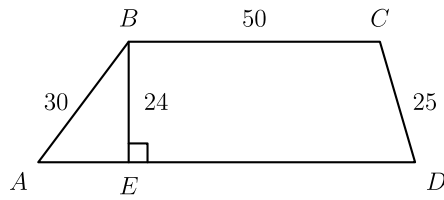We want to find how many k exist such that $13k$ is a three-digit number.

The smallest possible k such that $13k > 99$ is $k = 8$.

The largest possible k such that $13k < 1000$ is $k = 76$.

This tells us that k can range from 8 to 76, which gives us 69 values for k.

Thus, **C** is the correct answer.

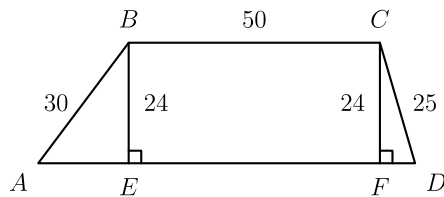**19.** What is the perimeter of trapezoid $ABCD$?

A	180

B	188

C	196

D	200

E	204

Solution(s):

Drop the altitude at C to intersect $\overline{AD}$ at F.

Then we can find $\overline{AE}$ and $overline{FD}$ using the Pythagorean theorem. We get that

$$AE = \sqrt{30^2 - 24^2}$$

$$= \sqrt{324}$$

$$= 18$$

and

$$DF = \sqrt{25^2 - 24^2}$$

$$= \sqrt{49}$$

$$= 7.$$

We also know that

$$EF = BC = 50.$$

The perimeter is therefore

$$50 + 30 + 25 + 18 + 7 + 50$$

$$= 180.$$

Thus, **A** is the correct answer.

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20. Alice and Bob play a game involving a circle whose circumference is divided by 12 equally-spaced points. The points are numbered clockwise, from 1 to 12. Both start on point 12. Alice moves clockwise and Bob, counterclockwise. In a turn of the game, Alice moves 5 points clockwise and Bob moves 9 points counterclockwise. The game ends when they stop on the same point. How many turns will this take?

A 6

B 8

C 12

D 14

E 24

Solution(s):

Let k be the number of turns. Then Alice moves $5k$ points and bob moves $9k$ points.

They coincide when combined, they have made a whole number of rotations. Namely, when $5k + 9k = 14k$ is a multiple of 12.

14 is a multiple of 2, so k must have must have a multiple of 2 and of 3. Therefore the smallest value of k that works is $2 \cdot 3 = 6$.

Thus, **A** is the correct answer.

21. How many distinct triangles can be drawn using three of the dots below as vertices?

• • •

• • •
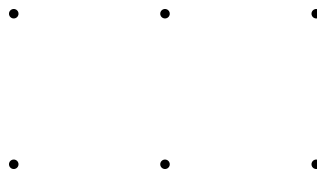
A 9

B 12

C 18

D 20

E 24

Solution(s):

Notice that choosing any of these 3 dots forms a triangle, except for the 2 triples that form a straight line.

There are $\binom{6}{3} = 20$ ways to choose the points, and then we subtract 2 to get $20 - 2 = 18$.

Thus, **C** is the correct answer.

22. A company sells detergent in three different sized boxes: small (S), medium (M) and large (L). The medium size costs 50% more than the small size and contains 20% less detergent than the large size. The large size contains twice as much detergent as the small size and costs 30% more than the medium size. Rank the three sizes from best to worst buy.

A SML

B LMS

C MSL

D LSM

E MLS

Solution(s):

WLOG, let the price of the small be $\$ 1$ and the size of the large be 10 oz.

Using these values, we get that the medium costs $\$ 1.50$ and contains 8 oz. The small's size is 5 **oz** and the large costs $\$ 1.95$.
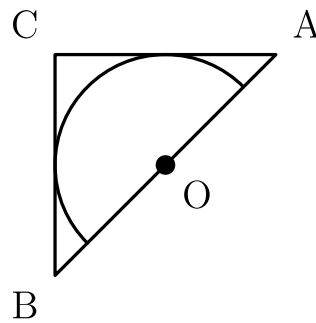
Calculating the unit prices, we get the small, medium, and large are

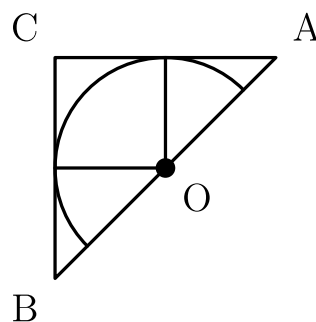$$0.200, 0, 188, \text{ and } 0.195$$

dollars per oz.

Thus, **E** is the correct answer.

23. Isosceles right triangle ABC encloses a semicircle of area 2π. The circle has its center O on hypotenuse $\overline{AB}$ and is tangent to sides $\overline{AC}$ and $\overline{BC}$. What is the area of triangle ABC?

C A
B

A	6
B	8
C	3π
D	10
E	4π

Solution(s):

C A
O
B

Note that the area of the full circle would be $2 \cdot 2\pi = 4\pi$. Then the radius r of the semicircle is

$$\sqrt{\frac{4\pi}{\pi}} = \sqrt{4} = 2.$$

We can also see that each leg of $\triangle ABC$ is two times r, which is $2 \cdot 2 = 4$.

Then the area of the triangle is

$$\frac{1}{2} \cdot 4 \cdot 4 = 8.$$

Thus, **B** is the correct answer.

24. A certain calculator has only two keys [+1] and [×2]. When you press one of the keys, the calculator automatically displays the result. For instance, if the calculator originally displayed "9" and you pressed [+1], it would display "10." If you then pressed [×2], it would display "20." Starting with the display "1," what is the fewest number of keystrokes you would need to reach "200"?

A	8
B	9
C	10
D	11
E	12

Solution(s):

We can work down from 200 to 1. If we are at even number, we can just divide by 2, and if it is odd, we subtract 1 before dividing.

$$200 \div 2 = 100$$
$$100 \div 2 = 50$$
$$50 \div 2 = 25$$
$$25 - 1 = 24$$
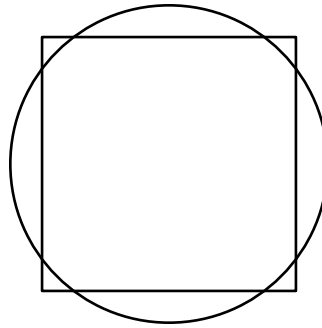$$24 \div 2 = 12$$
$$12 \div 2 = 6$$
$$6 \div 2 = 3$$
$$3 - 1 = 2$$
$$2 \div 2 = 1$$

From this, we can see that we need 9 key strokes.

Thus, **B** is the correct answer.

25. A square with side length 2 and a circle share the same center. The total area of the regions that are inside the circle and outside the square is equal to the total area of the regions that are outside the circle and inside the square. What is the radius of the circle?

A $\dfrac{2}{\sqrt{\pi}}$

B $\dfrac{1 + \sqrt{2}}{2}$

C $\dfrac{3}{2}$

D $\sqrt{3}$

E $\sqrt{\pi}$

Solution(s):

Let the area of the region inside the circle and outside the square be x. Then

$$4 - x = \pi r^2 - x.$$

This gives us that $\dfrac{4}{\pi} = r^2$. Then $r = \dfrac{2}{\sqrt{\pi}}$.

Thus, **A** is the correct answer.

Problems: https://live.poshenloh.com/past-contests/amc8/2005