2008 AMC 8 Solutions

1. Susan had $50 to spend at the carnival. She spent $12 dollars on food and twice as much on rides. How many dollars did she have left to spend?

A	12
B	14
C	26
D	38
E	50

    ### Solution(s):

    Susan spent $2 \cdot \$12 = \$24$ on rides. This means she spent a total of $12 + $24 = $36 at the carnival.

    This means that she has $50 - $36 = $14 left to spend.

    Thus, the answer is **B**.

2. The ten-letter code BEST OF LUCK represents the ten digits $0 - 9$, in order. What 4-digit number is represented by the code word CLUE?

A   8671

B   8672

C   9781

D   9782

E   9872

Solution(s):

The letter C is in the 9th position, so it would be the letter 8. The letter L is in the 7th position, so it would be the letter 6. The letter U is in the 8th position, so it would be the letter 7. The letter E is in the 2nd position, so it would be the letter 1.

Therefore, when putting together the word CLUE, we get 8671.

Thus, the answer is **A**.

3. If February is a month that contains Friday the 13th, what day of the week is February 1?

A	Sunday

B	Monday

C	Wednesday

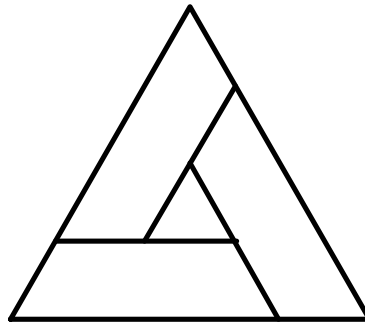| D	Thursday

E	Saturday

Solution(s):

Since the 13th is a Friday, we know that the 6th is also a Friday. The 1st is 5 days before the Friday, making it a Sunday.

Thus, the answer is **A**.

4. In the figure, the outer equilateral triangle has area 16, the inner equilateral triangle has area 1, and the three trapezoids are congruent. What is the area of one of the trapezoids?

A	3
B	4
C	5
D	6
E	7

Solution(s):

Since the area of the larger triangle is 16 and the area of the smaller triangle is 1. Thus, the area of the other 3 trapezoids is 15. Since the 3 trapezoids have a combined area of 15, each of their areas is $\dfrac{15}{3} = 5$.

Thus, the answer is **C**.

5. Barney Schwinn notices that the odometer on his bicycle reads 1441, a palindrome, because it reads the same forward and backward. After riding 4 more hours that day and 6 the next, he notices that the odometer shows another palindrome, 1661. What was his average speed in miles per hour?
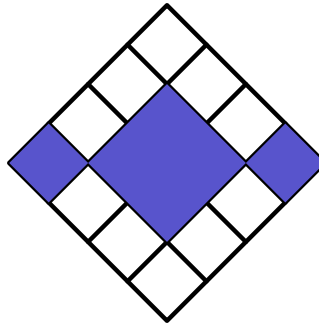
A   15

B   16

C   18

D   20

E   22

Solution(s):

The total distance traveled is $1661 - 1441 = 220$ miles. He also travelled 10 hours. Thus, the average speed is $\dfrac{220}{10} = 22$ miles per hour.

Thus, the answer is **E**.

6. In the figure, what is the ratio of the area of the colored squares to the area of the uncolored squares?

A	$3 : 10$
B	$3 : 8$
C	$3 : 7$
D	$3 : 5$
E	$1 : 1$

Solution(s):

The total area of the entire square is 16 since it is a 4×4 square. The area of the colored region is 6, making the remaining area $16 - 6 = 10$. Thus, the ratio is $6 : 10 = 3 : 5$.

Thus, the answer is **D**.

7. If $\dfrac{3}{5} = \dfrac{M}{45} = \dfrac{60}{N}$, what is $M + N$?

A   27

B   29

C   45

D   105

E   127

Solution(s):

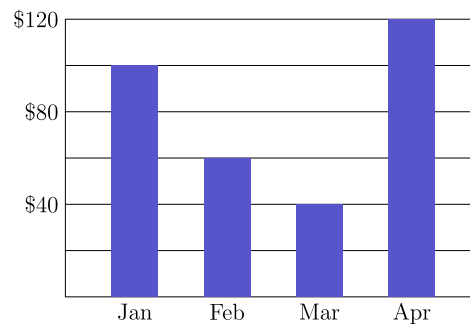$\dfrac{3}{5} = \dfrac{M}{45}$ means

$$M = \frac{3 \cdot 45}{5} = 27.$$

$\dfrac{3}{5} = \dfrac{60}{N}$ means

$$N = \frac{5 \cdot 60}{3} = 100.$$

Therefore, $M + N = 127$.

Thus, the answer is **E**.

8. Candy sales from the Boosters Club from January through April are shown. What were the average sales per month in dollars?

A   60

B   70

C   75

D   80

E   85

Solution(s):

The total sales are

$$100 + 60 + 40 + 120 = 320.$$

The average sales is then $\dfrac{320}{4} = 80$.

Thus, the answer is **D**.

9. In 2005 Tycoon Tammy invested 100 dollars for two years. During the first year her investment suffered a 15% loss, but during the second year the remaining investment showed a 20% gain. Over the two-year period, what was the change in Tammy's investment?

A $\quad$ 5% loss

B $\quad$ 2% loss

C $\quad$ 1% gain

D $\quad$ 2% gain

E $\quad$ 5% gain

Solution(s):

The 15% loss means the investment went from $\$100$ to $0.85 \cdot \$100 = \$85$.

The 20% gain means the investment went from $\$85$ to \$$85 \cdot 1.2 = \$102$. The total investment went from $\$100$ to $\$102$, making a gain of 2%.

Thus, the answer is **D**.

10. The average age of the 6 people in Room A is 40. The average age of the 4 people in Room B is 25. If the two groups are combined, what is the average age of all the people?

A   32.5

B   33

C   33.5

D   34

E   35

Solution(s):

The sum of the ages in Room A is

$$6 \cdot 40 = 240.$$

The sum of the ages in Room B is

$$4 \cdot 25 = 100.$$

The total sum is

$$240 + 100 = 340.$$

The average is therefore

$$\frac{340}{10} = 34.$$

Thus, the answer is **D**.

11. Each of the 39 students in the eighth grade at Lincoln Middle School has one dog or one cat or both a dog and a cat. Twenty students have a dog and 26 students have a cat. How many students have both a dog and a cat?

A   7

B   13

C   19

D   39

E   46

Solution(s):

The number of people that have both animals is equal to the number of people that own a cat plus the number of people that own a dog minus the number of people that own either.

Therefore, the number of people who own both is

$$20 + 26 - 39 = 7.$$

Thus, the answer is **A**.

12. A ball is dropped from a height of 3 meters. On its first bounce it rises to a height of 2 meters. It keeps falling and bouncing to $\frac{2}{3}$ of the height it reached in the previous bounce. On which bounce will it not rise to a height of 0.5 meters?

A	3

B	4

C	5

D	6

E	7

Solution(s):

Since the height of a bounce decreases by $\frac{2}{3}$ each bounce, the height of each bounce is $3 \cdot \left(\dfrac{2}{3}\right)^n$.

After 4 bounces, the ball bounces

$$3 \cdot \left(\frac{2}{3}\right)^4 = \frac{16}{27},$$

which is greater than $\frac{1}{2}$.

After 5 bounces, the ball bounces

$$3 \cdot \left(\frac{2}{3}\right)^5 = \frac{32}{81},$$

which is less than $\frac{1}{2}$.

Therefore, we are under $\frac{1}{2}$ after 5 bounces.

Thus, the answer is **C**.

13. Mr. Harman needs to know the combined weight in pounds of three boxes he wants to mail. However, the only available scale is not accurate for weights less than 100 pounds or more than 150 pounds. So the boxes are weighed in pairs in every possible way. The results are 122, 125 and 127 pounds. What is the combined weight in pounds of the three boxes?

A   160

B   170

C   187

D   195

E   354

Solution(s):

Let the weights be a, b, c. We know

$$\begin{cases} a + b = 122 \\ a + c = 125 \\ b + c = 127. \end{cases}$$

Adding all of this yields

$$2(a + b + c) =$$

$$122 + 125 + 127 = 374.$$

This makes

$$a + b + c = 187.$$

Thus, the answer is **C**.

14. Three A's, three B's, and three C's are placed in the nine spaces so that each row and column contain one of each letter. If A is placed in the upper left corner, how many arrangements are possible?

A		

A 2

B 3

C 4

D 5

E 6

Solution(s):

There are 2 rows and 2 columns to put B in, so there are 4 places for it. After that, there is 1 column and 1 row available for C, so the number of combinations is

$$2 \cdot 2 \cdot 1 \cdot 1 = 4.$$

Thus, the answer is **C**.

15. In Theresa's first 8 basketball games, she scored $7, 4, 3, 6, 8, 3, 1$ and 5 points. In her ninth game, she scored fewer than 10 points and her points-per-game average for the nine games was an integer. Similarly in her tenth game, she scored fewer than 10 points and her points-per-game average for the 10 games was also an integer. What is the product of the number of points she scored in the ninth and tenth games?

A 35

B 40

C 48

D 56

E 72

Solution(s):

The sum of her first 8 scores is 37. Since the average of the first 9 scores is an integer, the sum of the first 9 scores is a multiple of 9.

Since the score is less than 10, the sum of the scores after 9 games is between 37 and 47, and is a multiple of 9, making the sum 45. Thus, the score of the 9th game is $45 - 37 = 8$.
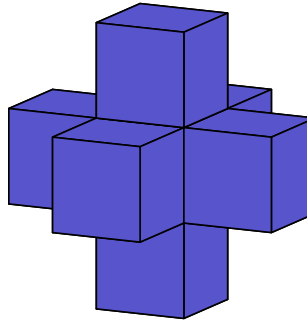
The sum of Theresa's first 9 scores is 45. Since the average of the first 10 scores is an integer, the sum of the first 10 scores is a multiple of 10.

Since the score is less than 10, the sum of the scores after 10 games is between 45 and 55, and is a multiple of 10, making the sum 50. Thus, the score of the 10th game is $50 - 45 = 5$.

Therefore, their product is $8 \cdot 5 = 40$.

Thus, the answer is **B**.

16. A shape is created by joining seven unit cubes, as shown. What is the ratio of the volume in cubic units to the surface area in square units?

A	$1 : 6$
B	$7 : 36$
C	$1 : 5$
D	$7 : 30$
E	$6 : 25$

Solution(s):

There are 7 unit cubes, so the volume is 7.

If we look at the perspective of the inner cube, we can see that there is one cube connected to each side. Furthermore, there are 5 exposed on each of the outer cubes. This makes the surface area $5 \cdot 6 = 30$.

This makes the ratio of volume to surface area $7 : 30$.

Thus, the answer is **D**.

17. Ms.Osborne asks each student in her class to draw a rectangle with integer side lengths and a perimeter of 50 units. All of her students calculate the area of the rectangle they draw. What is the difference between the largest and smallest possible areas of the rectangles?

A	76
B	120
C	128
D	132
E	136

Solution(s):

Let the length and width be l, w respectively. The rectangle would have an area of

$$l + r + l + r = 2(l + r) = 50.$$

Thus, $l + r = 25$, so $r = 25 - l$. The area is

$$lr = l(25 - l)$$

$$= 156.25 - (l - 12.5)^2.$$

This makes the area largest when l is as close as possible to 12.5 and the area is the smallest when l is as far from 12.5. Therefore, the largest area is when $l = 12, 13$ and the smallest area is when $l = 1, 24$.

This makes the larger area equal to

$$156.25 - (13 - 12.5)^2$$

$$= 156.25 - 0.25 = 156$$
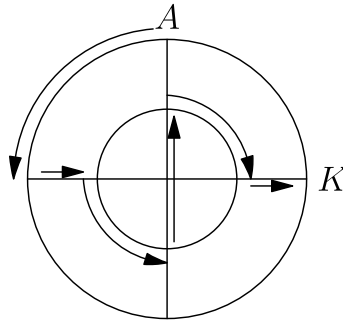
and the smaller area equal to

$$156.25 - (24 - 12.5)^2$$

$$= 156.25 - 132.25 = 24.$$

Thus, the difference is $156 - 24 = 132$.

Thus, the answer is **D**.

18. Two circles that share the same center have radii 10 meters and 20 meters. An aardvark runs along the path shown, starting at A and ending at K. How many meters does the aardvark run?

A	$10\pi + 20$

B	$10\pi + 30$

C	$10\pi + 40$

D	$20\pi + 20$

E	$20\pi + 40$

Solution(s):

The circumference of a circle is $\pi d = 2r\pi$, so going a quarter of the way around is $\dfrac{\pi r}{2}$.

He goes a quarter of the way around the large circle, so this part is 10π meters. He then goes from the larger circle to the smaller circle, which is $20 - 10 = 10$ meters.

He goes a quarter of the way around the smaller circle, so this part is 5π meters. He then goes through the diameter of the smaller circle, which is $10 \cdot 2 = 20$ meters.
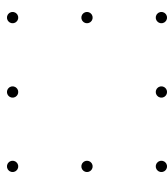
He then goes a quarter of the way around the smaller circle, so this part is 5π meters. He finally goes from the smaller circle to the larger circle, which is $20 - 10 = 10$ meters.

The total length is

$$10\pi + 10 + 5\pi + 20 + 5\pi + 10$$

$$= 20\pi + 40.$$

Thus, the answer is **E**.

19. Eight points are spaced around at intervals of one unit around a 2×2 square, as shown. Two of the 8 points are chosen at random. What is the probability that the two points are one unit apart?

A	$\dfrac{1}{4}$
B	$\dfrac{2}{7}$
C	$\dfrac{4}{11}$
D	$\dfrac{1}{2}$
E	$\dfrac{4}{7}$

Solution(s):

Each dot has 2 dots that are one unit away from it.

Therefore, regardless of the choice of the first dot, 2 of the other 7 dots would be within one unit, so the probability is $\dfrac{2}{7}$.

Thus, the answer is **B**.

20. The students in Mr. Neatkin's class took a penmanship test. Two-thirds of the boys and $\frac{3}{4}$ of the girls passed the test, and an equal number of boys and girls passed the test. What is the minimum possible number of students in the class?

A   12

B   17

C   24

D   27

E   36

Solution(s):

Since the number of boys who passed and the number of girls who passed are the same, we can assign them the same variable. Let this number be p.
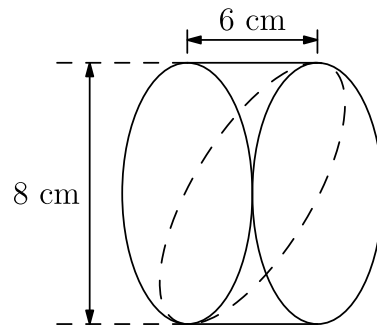
The number of boys in the class is $\frac{3}{2}p$, and the number of girls in the class is $\frac{4}{3}p$.

Thus, the total number of people is $\frac{4}{3}p + \frac{3}{2}p = \frac{17}{6}p$.

The total number of people must be a multiple of the numerator of this fraction, so the number of people must be a multiple of 17, making the number of people 17.

Thus, the answer is **B**.

21. Jerry cuts a wedge from a 6-cm cylinder of bologna as shown by the dashed curve. Which answer choice is closest to the volume of his wedge in cubic centimeters?

A   48

B   75

C   151

D   192

E   603

Solution(s):

The total volume is Ah where A is the area of the base and h is the height.

To find A, we take the area of the circle. It has a diameter of 8cm, so it has a radius of area 4cm. Thus, the area is $r^2\pi = 16\pi$. Since $h = 6$cm, the volume of the whole thing is $16\pi \cdot 6 = 96\pi$.

Since the wedge is half of the volume of the cylinder, the volume of the wedge is 48π.

To estimate this, we can find that

$$49\pi \approx 49 \cdot \frac{22}{7} = 154,$$

so

$$48\pi \approx 154 - \pi \approx 154 - 3 = 151.$$

Thus, the answer is **C**.

22. For how many positive integer values of n are both $\dfrac{n}{3}$ and $3n$ three-digit whole numbers?

A   12

B   21

C   27

D   33

E   34

Solution(s):

Let $x = \dfrac{n}{3}$. We know x and $9\dfrac{n}{3} = 9x$ are both three digit numbers.
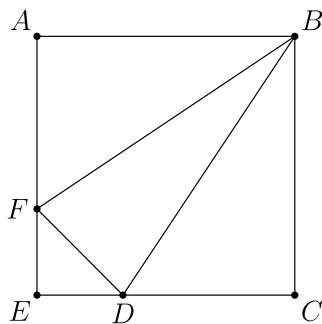
Thus,

$$100 \leq x, 9x \leq 999,$$

so

$$100 \leq x \leq 111.$$

Every integer in this range has x that creates a valid n, so there are 12 valid numbers.

Thus, the answer is **A**.

23. In square $ABCE$, $AF = 2FE$ and $CD = 2DE$. What is the ratio of the area of $\triangle BFD$ to the area of square $ABCE$?

A	$\dfrac{1}{6}$
B	$\dfrac{2}{9}$
C	$\dfrac{5}{18}$
D	$\dfrac{1}{3}$
E	$\dfrac{7}{20}$

Solution(s):

Let the side length be s. Note that the total area is s^2.

Since $AF = 2FE$, we know

$$FE = \frac{AF}{3} = \frac{s}{3},$$

$$AF = 2\frac{AE}{3} = \frac{2s}{3}.$$

Since $CD = 2DE$, we know

$$CD = \frac{CE}{3} = \frac{s}{3},$$

$$AF = 2\frac{AE}{3} = \frac{2s}{3}.$$

This makes the area of

$$ABF = \frac{s \cdot \frac{2s}{3}}{2} = \frac{s^2}{3},$$

the area of

$$BCD = \frac{s \cdot \frac{2s}{3}}{2} = \frac{s^2}{3},$$

and the area of

$$FED = \frac{\frac{s}{3} \cdot \frac{s}{3}}{2} = \frac{s^2}{18}.$$

Thus, the area of

$$BFD = s^2 - \frac{s^2}{3} - \frac{s^2}{3} - \frac{s^2}{18}$$

$$= \frac{5s^2}{18}.$$

Thus, the answer is **C**.

24. Ten tiles numbered 1 through 10 are turned face down. One tile is turned up at random, and a die is rolled. What is the probability that the product of the numbers on the tile and the die will be a square?

A $\dfrac{1}{10}$

B $\dfrac{1}{6}$

C $\dfrac{11}{60}$

D $\dfrac{1}{5}$

E $\dfrac{7}{30}$

Solution(s):

If the rolled number was 1, then the tile can be $1, 4, 9$, yielding 3 combinations.

If the rolled number was 2, then the tile can be $2, 8$, yielding 2 combinations.

If the rolled number was 3, then the tile can be 3, yielding 1 combination.

If the rolled number was 4, then the tile can be $1, 4, 9$, yielding 3 combinations.

If the rolled number was 5, then the tile can be 5, yielding 1 combination.

If the rolled number was 6, then the tile can be 6, yielding 1 combination.

The total number of combinations is

$$3 + 2 + 1 + 3 + 1 + 1 = 11.$$

There are 60 combinations each with equal likelihood, so the probability is $\dfrac{11}{60}$.

Thus, the answer is **C**.

25. Margie's winning art design is shown. The smallest circle has radius 2 inches, with each successive circle's radius increasing by 2 inches. Which of the following is closest to the percent of the design that is dark-colored?

A	42
B	44
C	45
D	46
E	48

Solution(s):

The largest circle is of radius 12, so the entire design has area of $12^2\pi = 144\pi$.

Each dark region is the area of its circle minus the area of the previous circle. The largest dark area would be of area

$$(10^2 - 8^2)\pi = 36\pi,$$

the next area would be

$$(6^2 - 4^2)\pi = 20\pi,$$

and the smallest area would be

$$(2^2 - 0^2)\pi = 4\pi.$$

Therefore, the combined dark area would be 60π.

This fraction would be $\dfrac{60\pi}{144\pi} = \dfrac{5}{12}$, which is approximately 42%.

Thus, the answer is **A**.

Problems: https://live.poshenloh.com/past-contests/amc8/2008