2010 AMC 8 Solutions

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1. At Euclid Middle School the mathematics teachers are Mrs. Germain, Mr. Newton, and Mrs. Young. There are 11 students in Mrs. Germain's class, 8 students in Mr. Newton's class, and 9 students in Mrs. Young's class taking the AMC 8 this year. How many mathematics students at Euclid Middle School are taking the contest?

A 26

B 27

C 28

D 29

E 30

Solution(s):

There are $11+8+9=28$ students.

Therefore, the answer is **C**.

2. If $a * b = \dfrac{a \times b}{a + b}$ for a, b positive integers, then what is $5 * 10$?

A $\dfrac{3}{10}$

B 1

C 2

D $\dfrac{10}{3}$

E 50

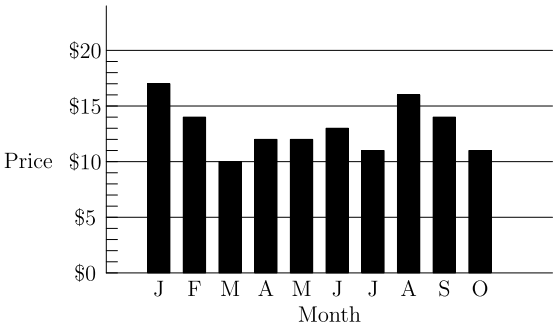
Solution(s):

Given our definition of $a * b$, we have

$$5 * 10 = \frac{5 \times 10}{5 + 10} = \frac{50}{15} = \frac{10}{3}.$$

Thus, the answer is **D**.

3. The graph shows the price of five gallons of gasoline during the first ten months of the year. By what percent is the highest price more than the lowest price?



A 50

B 62

C 70

D 89

E 100

Solution(s):

The highest price is 17 and the lowest price is 10. This means the percent is

$$(\frac{17}{10}-1)*100=70.$$

Thus, the answer is **C**.

4. What is the sum of the mean, median, and mode of the numbers $2, 3, 0, 3, 1, 4, 0, 3$?

A 6.5

B 7

C 7.5

D 8.5

E 9

Solution(s):

The list reordered is $0, 0, 1, 2, 3, 3, 3, 4$.

The median is the mean of the middle two numbers, which would be $\frac{2+3}{2} = 2.5$.

The mode is 3 since 3 appears the most. The mean is

$$\frac{0+0+1+2+3+3+3+4}{8}$$

$$= \frac{16}{8} = 2.$$

Their sum is $2.5 + 3 + 2 = 7.5$.

Thus, their answer is **C**.

5. Alice needs to replace a light bulb located 10 centimeters below the ceiling in her kitchen. The ceiling is 2.4 meters above the floor. Alice is 1.5 meters tall and can reach 46 centimeters above the top of her head. Standing on a stool, she can just reach the light bulb. What is the height of the stool, in centimeters?

A 32

B 34

C 36

D 38

E 40

Solution(s):

We know the height of the ceiling is $2.4 \cdot 100 = 240$ cm. Subtracting out all the given values, we get

$$240 - 150 - 46 - 10 = 34.$$

Therefore, the height of the stool is 34 cm. Thus, **B** is the correct answer.

6. Which of the following figures has the greatest number of lines of symmetry?

A equilateral triangle

B non-square rhombus

C non-square rectangle

D isosceles trapezoid

E square

Solution(s):

First, each line of symmetry must go through the center of the shape. This would ensure that the center of the shape isn't on only one side, as that would make it asymmetric.

Next, if a line goes through any side, it must go through its midpoint, to ensure that after a reflection over this line, the same amount of the line is on both sides. Moreover, it must be perpendicular, to ensure that the line when reflected stays on itself.

Similarly, if a line goes through any corner, it must go through its angle bisector, to ensure that after a reflection over this line, the angle of the line is the same after reflection. Moreover, it must have the same side length on both sides.

Now, lets look at each of the shapes. An equilateral triangle has 3 lines that intersect a corner or a midpoint, so it has at most 3 symmetry lines. A non-square rhombus only has two symmetry lines, as only the lines that go through the corners work, but not the ones through the midpoints as they would not intersect perpendicularly. A non-square rectangles only has two symmetry lines as it has symmetry lines through the midpoints of opposite sides, but not through the corners since it doesn't bisect the angle. An isosceles trapezoid only has 1 symmetry line, that goes through the midpoints of opposite sides. A square has 4 symmetry lines, which go through opposite corners and opposite midpoints.

A square therefore has the most symmetry lines.

Thus, the answer is **E**.

7. Using only pennies, nickels, dimes, and quarters, what is the smallest number of coins Freddie would need so he could pay any amount of money less than a dollar?

A 6

B 10

C 15

D 25

E 99

Solution(s):

Freddie needs 4 pennies as this is the only way to pay 4 cents. Next, he needs 1 nickel or 5 pennies to make a 9 cents after having the 4 starting pennies. Here, we have 5 coins. Using 1 nickel minimizes the coins needed. Now, to make 24 cents, we need 15 more cents. This can be done with a dime and nickel. Now, we have 7 coints Now, for the last 75 cents, we can use 3 quarters to make the rest. This leaves us with 10 coins.

Therefore, the answer is **B**.

8. As Emily is riding her bicycle on a long straight road, she spots Emerson skating in the same direction $1/2$ mile in front of her. After she passes him, she can see him in her rear mirror until he is $1/2$ mile behind her. Emily rides at a constant rate of 12 miles per hour, and Emerson skates at a constant rate of 8 miles per hour. For how many minutes can Emily see Emerson?

A 6

B 8

C 12

D 15

E 16

Solution(s):

Let d be how far Emily is ahead of Emerson. Emily sees Emerson if $-\frac{1}{2} \leq d \leq \frac{1}{2}$. Suppose at $t = 0$, where t is in hours, that $d = -\frac{1}{2}$. Then, $d = (12 - 8)t - \frac{1}{2}$. Since we must find where $d = \frac{1}{2}$, we find the time where

$$\frac{1}{2} = 4t - \frac{1}{2} \implies 1 = 4t$$

$$\implies t = 0.25.$$

Since 0.25 hours passed, we know that $60 * 0.25 = 15$ minutes passed.

Therefore, the answer is **D**.

9. Ryan got 80% of the problems correct on a 25-problem test, 90% on a 40-problem test, and 70% on a 10-problem test. What percent of all the problems did Ryan answer correctly?

A 64

B 75

C 80

D 84

E 86

Solution(s):

There were a total of $25 + 40 + 10 = 75$ problems.

On the first test, he solved $0.8 * 25 = 20$ problems.

On the second test, he solved $0.9 * 40 = 36$ problems.

On the third test, he solved $0.7 * 10 = 7$ problems.

Therefore, he solved a total of $20 + 36 + 7 = 63$. This means the fraction he solved is $\frac{63}{75} = 84\%$.

Therefore, the answer is **D**.

10. Six pepperoni circles will exactly fit across the diameter of a 12-inch pizza when placed. If a total of 24 circles of pepperoni are placed on this pizza without overlap, what fraction of the pizza is covered by pepperoni?

A $\frac{1}{2}$

B $\frac{2}{3}$

C $\frac{3}{4}$

D $\frac{5}{6}$

E $\frac{7}{8}$

Solution(s):

Each circle has $\frac{1}{6}$ the diameter of the large circle, so it has $(\frac{1}{6})^2 = \frac{1}{36}$ of the total area.

Since there are 24 pepperoni, they take up $24 * \frac{1}{36} = \frac{2}{3}$ of the area.

Therefore, the answer is **B**.

11. The top of one tree is 16 feet higher than the top of another tree. The heights of the two trees are in the ratio $3:4$. In feet, how tall is the taller tree?

A 48

B 64

C 80

D 96

E 112

Solution(s):

Let b, s be the heights of bigger and smaller trees respectively. Then, $b = s + 16$ and $s = 0.75b$. If we substitute, we get

$$b = 0.75b + 16 \implies 0.25b = 16$$

$$\implies b = 64.$$

Thus, the answer is **B**.

12. Of the 500 balls in a large bag, 80% are red and the rest are blue. How many of the red balls must be removed so that 75% of the remaining balls are red?

A 25

B 50

C 75

D 100

E 150

Solution(s):

If r is the number of red balls, then

$$r = 0.8 * 500 = 400.$$

Therefore, if b is the number of blue balls, then

$$b = 500 - 400 = 100.$$

If there are 75 red balls after removing balls, then there are 25 blue balls. This means the total number of balls is $\frac{100}{.25} = 400$. This means the total number of balls decreased by $500 - 400 = 100$.

Thus, the answer is **D**.

13. The lengths of the sides of a triangle in inches are three consecutive integers. The length of the shortest side is 30% of the perimeter. What is the length of the longest side?

A 7

B 8

C 9

D 10

E 11

Solution(s):

Let s be the smallest length. Then, all the side lengths are $s, s+1, s+2$. This would make the perimeter equal to $3s+3$. Since $s = 0.3(3s+3)$, then $s = 0.9s + 0.9$. This makes $0.1s = 0.9$, so $s = 9$ which makes the longest side length $s+2 = 11$.

Thus, the answer is **E**.

14. What is the sum of the prime factors of 2010?

A 67

B 75

C 77

D 201

E 210

Solution(s):

First, the primes $2, 5$ are factors of 2010 since it is a multiple of 10. Dividing 2010 by 10 is 201. Then, 3 is a factor of 201 since the digit sum of 201 is a multiple of 3. Dividing this by 3 yields the prime number 67. This means the prime factors are $2, 3, 5, 67$, which makes their sum 77.

Thus, the correct answer is **C**.

15. A jar contains 5 different colors of gumdrops. 30% are blue, 20% are brown, 15% are red, 10% are yellow, and the other 30 gum drops are green. If half of the blue gumdrops are replaced with brown gumdrops, how many gumdrops will be brown?

A 35

B 36

C 42

D 48

E 64

Solution(s):

Since we have percentages for every color except green, the percent of green is 100% minus the sum of the other colors. This would make the percent of green equal to

$$100\% - 30\% - 20\% -$$

$$15\% - 10\% = 25\%$$

Since we know 30 jelly beans is 25%, we know that the total number of jelly beans is $\frac{30}{.25} = 120$.

This means there are $.2 * 120 = 24$ brown jelly beans to start and $.3 * 120 = 36$ blue jelly beans. If half of the blue jelly beans are turned to brown, then $\frac{36}{2} = 18$ more brown jelly beans are added. Therefore, we have $24 + 18 = 42$ brown jelly beans.

Thus, the answer is **C**.

16. A square and a circle have the same area. What is the ratio of the side length of the square to the radius of the circle?

A $\frac{\sqrt{\pi}}{2}$

B $\sqrt{\pi}$

C π

D 2π

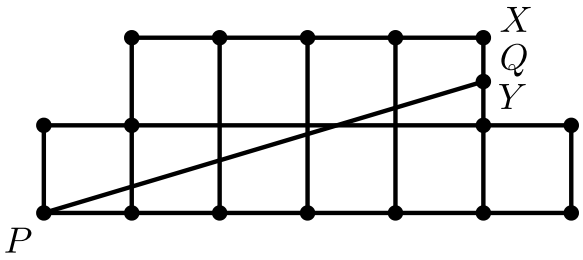
E π^2

Solution(s):

Let s be the side length of the square and let r be the radius of the circle. Then, since they have the same areas, $s^2 = \pi r^2$. This means $(\frac{s}{r})^2 = \pi$, so $\frac{s}{r} = \sqrt{\pi}$.

Thus, the answer is **B**.

17. The diagram shows an octagon consisting of 10 unit squares. The portion below $\overline{PQ}$ is a unit square and a triangle with base 5. If $\overline{PQ}$ bisects the area of the octagon, what is the ratio $\dfrac{XQ}{QY}$?



A $\dfrac{2}{5}$

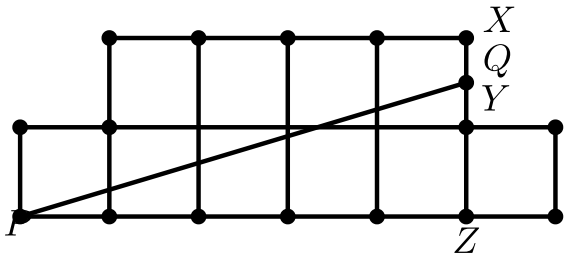
B $\dfrac{1}{2}$

C $\dfrac{3}{5}$

D $\dfrac{2}{3}$

E $\dfrac{3}{4}$

Solution(s):



Since PQ bisects the area, the area under the line is 5. Removing the square on the right makes the bottom a triangle of base 5 with area 4. Let the base of this triangle be PZ.

The area being 4 means

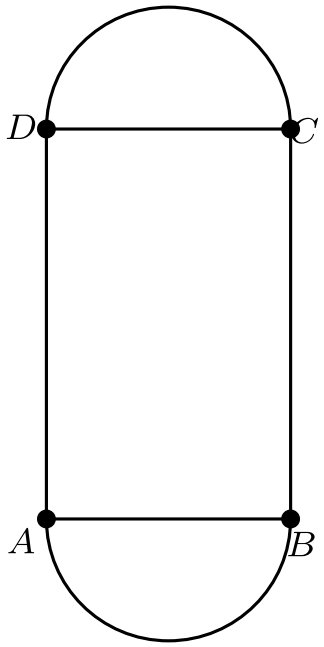
$$\frac{(PZ)(ZQ)}{2} = \frac{5(ZQ)}{2} = 4$$

$$\implies ZQ = 1.6.$$

Therefore, $QY = QZ - 1 = 0.6$ and $XQ = 2 - QZ = 0.4$. This would make $\frac{XQ}{QY} = \frac{0.4}{0.6} = \frac{2}{3}$.

Thus, the answer is **D**.

18. A decorative window is made up of a rectangle with semicircles at either end. The ratio of AD to AB is $3:2$. And AB is 30 inches. What is the ratio of the area of the rectangle to the combined area of the semicircles?



A $2:3$

B $3:2$

C $6:\pi$

D $9:\pi$

E $30:\pi$

Solution(s):

Combining the semicircles would make a circle of diameter $d = 30$. This would make the radius equal to $\frac{d}{2}$. Therefore, the combined area of the semicircles is

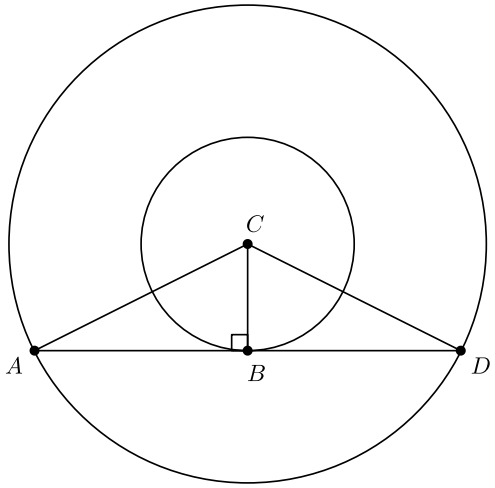
$$(\frac{d}{2})^2 * \pi = \frac{\pi d^2}{4}.$$

Side $AD = \frac{3}{2}$ and $AB = \frac{3}{2}d$. The area of the rectangle is therefore $\frac{3}{2}d * d = \frac{3}{2}d^2$.

The ratio of the area of the rectangle to the area of the semicircles is $\frac{\frac{3}{2}d^2}{\frac{\pi d^2}{4}} = \frac{6}{\pi}$.

Thus, the answer is **C**.

19. The two circles pictured have the same center C. Chord $\overline{AD}$ is tangent to the inner circle at B, AC is 10, and chord $\overline{AD}$ has length 16. What is the area between the two circles?



A 36π

B 49π

C 64π

D 81π

E 100π

Solution(s):

The area between the two circles is the area of the larger circle minus the area of the smaller circle. This would be

$$(AC)^2\pi - (CB)^2\pi$$

$$= \pi(AC^2 - CB^2).$$

By the Pythagorean Theorem, we can get

$$AC^2 - CB^2 = AB^2.$$

Therefore, we need to find $AB^2\pi$.

SInce AB is half of AD, we get $AB = 8$. This makes $AB^2\pi = 64\pi$.

Thus, the area is **C**.

20. In a room, $2/5$ of the people are wearing gloves, and $3/4$ of the people are wearing hats. What is the minimum number of people in the room wearing both a hat and a glove?

A 3

B 5

C 8

D 15

E 20

Solution(s):

Since our room has $\frac{2}{5}$ of the people wearing gloves, the number of people must be a multiple of 5. Since our room has $\frac{3}{4}$ of the people wearing hats, the number of people must be a multiple of 4. Therefore, the people in the room must be a multiple of 20.

Now, we can also use the following formula by the principle of inclusion exclusion: Fraction of people wearing both = Fraction of people wearing gloves + Fraction of people wearing hats - Fraction of people wearing either.

This makes our desired fraction equal to $\frac{2}{5}+\frac{3}{4}-$ Fraction of people who wear either. If we wish to minimize the number of who wear both, we maximize the number of people who wear either, to make it 1. Therefore, the fraction of people that wear both is $\frac{2}{5}+\frac{3}{4}-1=\frac{3}{20}$.

Since our number is a (positive) multiple of 20, we have the number of people wearing both as 3 if we choose to have just 20 people.

Therefore, **A** is the correct answer.

21. Hui is an avid reader. She bought a copy of the best seller Math is Beautiful. On the first day, Hui read $1/5$ of the pages plus 12 more, and on the second day she read $1/4$ of the remaining pages plus 15 pages. On the third day she read $1/3$ of the remaining pages plus 18 pages. She then realized that there were only 62 pages left to read, which she read the next day. How many pages are in this book?

A 120

B 180

C 240

D 300

E 360

Solution(s):

The pages left after the third day is 62. Before reading the last 18 pages, she had 80 pages left. This is $\frac{2}{3}$ of the pages remaining, so she had 120 pages left before the third day.

Before reading the 15 pages, she had $120 + 15 = 135$ pages left. This is $\frac{3}{4}$ of the pages remaining, so she had 180 pages left before the second day.

Before reading the 12 pages, she had $180 + 12 = 192$ pages left. This is $\frac{4}{5}$ of the pages remaining, so she had 240 pages left before the first day, making the book 240 pages.

Thus, the answer is **C**.

22. The hundreds digit of a three-digit number is 2 more than the units digit. The digits of the three-digit number are reversed, and the result is subtracted from the original three-digit number. What is the units digit of the result?

A 0

B 2

C 4

D 6

E 8

Solution(s):

Let the units digit be u, and let the tens digit be t. This makes the hundreds digit be $u+2$. This makes the number equal to

$$100(u+2)+10t+u$$

$$=201u+200+10t$$

and the reversed number is

$$100u+10t+(u+2)$$

$$=201u+2+10t.$$

This makes the difference equal to

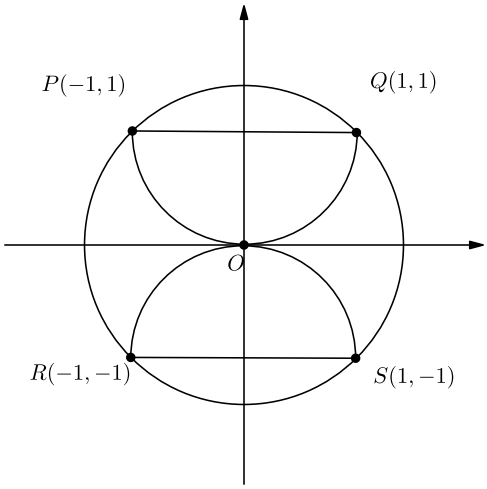
$$(201u+10t+200)-$$

$$(201u+10t+2)=198.$$

This makes the units digit 8.

Therefore, the units digit is **E**.

23. Semicircles POQ and ROS pass through the center O. What is the ratio of the combined areas of the two semicircles to the area of circle O?



A $\frac{\sqrt{2}}{4}$

B $\frac{1}{2}$

C $\frac{2}{\pi}$

D $\frac{2}{3}$

E $\frac{\sqrt{2}}{2}$

Solution(s):

The area of each of the smaller circles have an area of $\pi\frac{r^2}{2}$. Each of them have a radius of 1, so their combined area is $\pi\frac{1}{2}+\pi\frac{1}{2}=\pi$.

Next, the radius of the larger circle is equal to the length of OQ, which is equal to $\sqrt{1^2+1^2}=\sqrt{2}$. Its area is $\pi r^2=\pi(\sqrt{2})^2=2\pi$.

This means the ratio is $\frac{\pi}{2\pi}=\frac{1}{2}$.

Thus, the answer is **B**.

24. What is the correct ordering of the three numbers, 10^8, 5^{12}, and 2^{24}?

A 2^{24} < 10^8 < 5^{12}

B 2^{24} < 5^{12} < 10^8

C 5^{12} < 2^{24} < 10^8

D 10^8 < 5^{12} < 2^{24}

E 10^8 < 2^{24} < 5^{12}

Solution(s):

First, we get

$$2^{24} = (2^8)(4^8) < (2^8)(5^8) = 10^8.$$

Next, we get

$$10^8 = (2^8)(5^8) =$$

$$(4^4)(5^8) < (5^4)(5^8) = 5^{12}.$$

This means $2^{24} < 10^8 < 5^{12}$.

Thus, the answer is **A**.

25. Everyday at school, Jo climbs a flight of 6 stairs. Jo can take the stairs 1, 2, or 3 at a time. For example, Jo could climb 3, then 1, then 2. In how many ways can Jo climb the stairs?

A 13

B 18

C 20

D 22

E 24

Solution(s):

Lets count first the number of ways to climb a flight of stairs. She must land on the last stair, and for each of the other stairs, she can either step on it or not step on it. This means there are $2^5 = 32$ total combinations. Now, we must exclude the ways that include stepping more than 3 steps. This would mean we climb $4, 5$ or 6 stairs in one step. To do this, we can do it in the following ways:

$$6, 1-5, 5-1, 4-2, 2-4,$$

$$1-1-4, 1-4-1, 4-1-1.$$

This means we have 8 cases to exlude, so we have $32 - 8 = 24$ total valid cases.

Thus, the answer is **E**.

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