2014 AMC 8 Solutions

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1. Harry and Terry are each told to calculate $8-(2+5)$. Harry gets the correct answer. Terry ignores the parentheses and calculates $8-2+5$. If Harry's answer is H and Terry's answer is T, what is $H-T$?

A -10

B -6

C 0

D 6

E 10

Solution(s):

Harry calculates it correctly as follows:

$$\begin{aligned} 8-(2+5) &= 8-7 \\ &= 1. \end{aligned}$$

Terry calculates it incorrectly as follows:

$$\begin{aligned} 8-2+5 &= 6+5 \\ &= 11. \end{aligned}$$

Therefore:

$$\begin{aligned} H-T &= 1-11 \\ &= -10. \end{aligned}$$

Thus, **A** is the correct answer.

2. Paul owes Paula 35 cents and has a pocket full of 5-cent coins, 10-cent coins, and 25-cent coins that he can use to pay her. What is the difference between the largest and the smallest number of coins he can use to pay her?

A 1

B 2

C 3

D 4

E 5

Solution(s):

To use the most number of coins, Paul would have to use only 5-cent coins. This would require $35/5 = 7$ coins.

To use the least number of coins, Tom would use a 25-cent coin and a 10-cent coin, for a total of 2 coins.

Therefore, the difference is $7 - 2 = 5$.

Thus, **E** is the correct answer.

3. Isabella had a week to read a book for a school assignment. She read an average of 36 pages per day for the first three days and an average of 44 pages per day for the next three days. She then finished the book by reading 10 pages on the last day. How many pages were in the book?

A 240

B 250

C 260

D 270

E 280

Solution(s):

During the first three days, Isabella read $36 \cdot 3 = 108$ pages.

During the next three days, Isabella read $44 \cdot 3 = 132$ pages.

Therefore, she read a total of $108 + 132 + 10 = 250$ pages.

Thus, **B** is the correct answer.

4. The sum of two prime numbers is 85. What is the product of these two prime numbers?

A 85

B 91

C 115

D 133

E 166

Solution(s):

The only for two numbers to add to an odd number is if one is even and the other is odd. The only even prime is 2, so 2 is one of the prime numbers.

Therefore, the other prime is $85 - 2 = 83$, and the product is $2 \cdot 83 = 166$.

Thus, **E** is the correct answer.

5. Margie's car can go 32 miles on a gallon of gas, and gas currently costs \$\(4\) per gallon. How many miles can Margie drive on \(\$ 20\) worth of gas?

A 64

B 128

C 160

D 320

E 640

Solution(s):

With $\$20$, Margie can buy $20/4 = 5$ gallons of gas. With 5 gallons of gas, Margie can travel $5 \cdot 32 = 160$ miles.

Thus, **C** is the correct answer.

6. Six rectangles each with a common base width of 2 have lengths of $1, 4, 9, 16, 25,$ and 36. What is the sum of the areas of the six rectangles?

A 91

B 93

C 162

D 182

E 202

Solution(s):

To find the area of each rectangle we multiply 2 by their respective lengths. This means that we can factor out the 2 and we are left with the sum of the lengths. Adding together the lengths, we get 91.

Therefore, the sum of the areas is $2 \cdot 91 = 182$.

Thus, **D** is the correct answer.

7. There are four more girls than boys in Ms. Raub's class of 28 students. What is the ratio of number of girls to the number of boys in her class?

A $3:4$

B $4:3$

C $3:2$

D $7:4$

E $2:1$

Solution(s):

Let x be the number of boys in the class. This means that there are $x+4$ girls. As there are 28 students, we know that:

$$\begin{aligned} x+x+4 &= 28 \\ 2x &= 24 \\ x &= 12. \end{aligned}$$

Therefore, the ratio of girls to boys is $16:12=4:3$.

Thus, **B** is the correct answer.

8. Eleven members of the Middle School Math Club each paid the same amount for a guest speaker to talk about problem solving at their math club meeting. They paid their guest speaker $\$\underline{1A2}$. What is the missing digit A of this 3-digit number?

A 0

B 1

C 2

D 3

E 4

Solution(s):

Note that since 11 people paid the same amount, then the resulting sum is divisible by 11. Therefore, $\underline{1A2}$ must be divisible by 11.

Remember the divisibility rule for 11: if we take the difference of the sums of alternating digits, and this difference is divisible by 11, the whole number is divisible by 11.

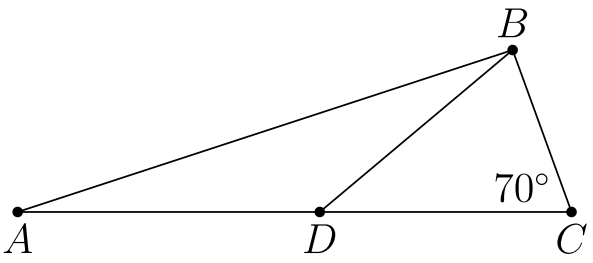
For $\underline{1A2}$, the aforementioned difference is

$$1 + 2 - A = 3 - A.$$

The only way for this to be divisible by 11 is if $A = 3$.

Thus, **D** is the correct answer.

9. In $\triangle ABC$, D is a point on side $\overline{AC}$ such that $BD = DC$ and $\angle BCD$ measures 70°. What is the degree measure of $\angle ADB$?



A 100

B 120

C 135

D 140

E 150

Solution(s):

Since $\triangle BDC$ is isosceles, we know that

$$\angle DBC = \angle DCB = 70^\circ.$$

This means that

$$\begin{aligned} \angle BDC &= 180^\circ - 70^\circ - 70^\circ \\ &= 40^\circ. \end{aligned}$$

Therefore,

$$\begin{aligned} \angle ADB &= 180^\circ - \angle BDC \\ &= 180^\circ - 140^\circ \\ &= 140^\circ. \end{aligned}$$

Thus, **D** is the correct answer.

10. The first AMC 8 was given in 1985 and it has been given annually since that time. Samantha turned 12 years old the year that she took the seventh AMC 8. In what year was Samantha born?

A 1979

B 1980

C 1981

D 1982

E 1983

Solution(s):

The seventh AMC 8 would have been administered 6 years after the first one. Therefore, Samantha took it in $1985 + 6 = 1991$.

This means that Samantha was born 12 years prior in $1991 - 12 = 1979$.

Thus, **A** is the correct answer.

11. Jack wants to bike from his house to Jill's house, which is located three blocks east and two blocks north of Jack's house. After biking each block, Jack can continue either east or north, but he needs to avoid a dangerous intersection one block east and one block north of his house. In how many ways can he reach Jill's house by biking a total of five blocks?

A 4

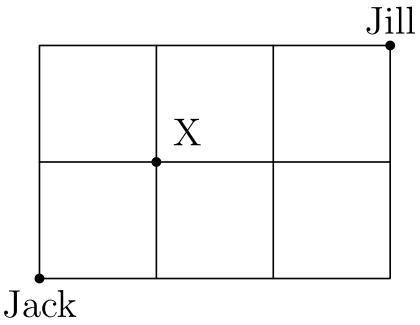
B 5

C 6

D 8

E 10

Solution(s):



Allow the X is the location of the dangerous intersection.

Let E represent moving one block east and N represent moving one block north. To avoid the dangerous intersection, Jack must go either EE or NN first.

After these moves, there are only 4 possible remaining options:

- $EEENN$
- $EENEN$
- $EENNE$
- $NNEEE$

Thus, **A** is the correct answer.

12. A magazine printed photos of three celebrities along with three photos of the celebrities as babies. The baby pictures did not identify the celebrities. Readers were asked to match each celebrity with the correct baby pictures. What is the probability that a reader guessing at random will match all three correctly?

A $\frac{1}{9}$

B $\frac{1}{6}$

C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{1}{2}$

Solution(s):

Notice that there are $3! = 6$ total ways that a reader could match the celebrities. However, only one of these is the correct matching. Therefore, the probability that the reader guesses it correctly is $\frac{1}{6}$.

Thus, **B** is the correct answer.

13. If n and m are integers and $n^2 + m^2$ is even, which of the following is impossible?

A n and m are even

B n and m are odd

C $n + m$ is even

D $n + m$ is odd

E none of these are impossible

Solution(s):

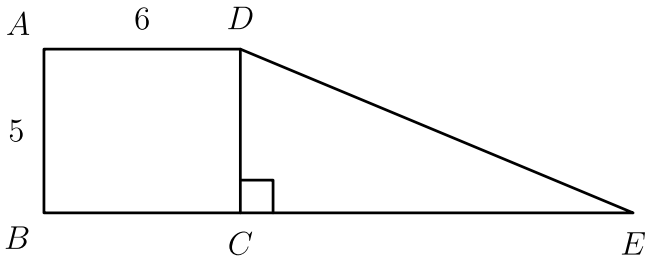
Since $n^2 + m^2$ is even, either n^2 and m^2 are both odd, or both even.

If they are both odd, then n and m are both odd. If they are both even, then n and m are both even. If n and m are both odd or even, their sum will always be even.

Therefore, $n + m$ is never odd.

Thus, **D** is the correct answer.

14. Rectangle $ABCD$ and right triangle DCE have the same area. They are joined to form a trapezoid, as shown. What is DE?



A 12

B 13

C 14

D 15

E 16

Solution(s):

The area of $ABCD$ is

$$5 \cdot 6 = 30.$$

The area of

$$\begin{aligned} \triangle DCE &= \frac{1}{2} DC \cdot CE \\ &= 30 \end{aligned}$$

Therefore:

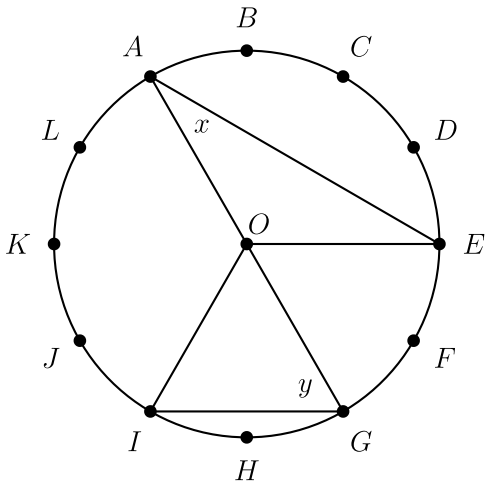
$$\begin{aligned} \frac{1}{2} \cdot 5 \cdot CE &= 30 \\ CE &= 12. \end{aligned}$$

Then using the Pythagorean theorem we get that

$$\begin{aligned} DE &= \sqrt{5^2 + 12^2} \\ &= \sqrt{169} \\ &= 13. \end{aligned}$$

Thus, **B** is the correct answer.

15. The circumference of the circle with center O is divided into 12 equal arcs, marked the letters A through L as seen below. What is the number of degrees in the sum of the angles x and y?



A 75

B 80

C 90

D 120

E 150

Solution(s):

Note that each of the 12 arcs splits the circle evenly, so they each cover $360^\circ/12 = 30^\circ$.

$\angle AOE$ spans 4 of these arcs, so

$$\angle AOE = 4 \cdot 30^\circ = 120^\circ.$$

Similarly,

$$\angle GOI = 2 \cdot 30^\circ = 60^\circ.$$

We also know that both triangles are isosceles since two of their sides are radii. Therefore,

$$x = \frac{180 - 120}{2} = 30^\circ$$

and

$$y = \frac{180 - 60}{2} = 60^\circ.$$

Therefore, $x + y = 90^\circ$.

Thus, **C** is the correct answer.

16. The "Middle School Eight" basketball conference has 8 teams. Every season, each team plays every other conference team twice (home and away), and each team also plays 4 games against non-conference opponents. What is the total number of games in a season involving the "Middle School Eight" teams?

A 60

B 88

C 96

D 144

E 160

Solution(s):

Each time plays 4 non-conference games, totalling $4 \cdot 8 = 32$ games.

We don't want to double count the conference game, so we can just count all the games every team played out home. Each team played 7 home games, for a total of $8 \cdot 7 = 56$ home games.

Therefore, the total number of games in a season is

$$32 + 56 = 88.$$

Thus, **B** is the correct answer.

17. George walks 1 mile to school. He leaves home at the same time each day, walks at a steady speed of 3 miles per hour, and arrives just as school begins.

Today he was distracted by the pleasant weather and walked the first $\frac{1}{2}$ mile at a speed of only 2 miles per hour. At how many miles per hour must George run the last $\frac{1}{2}$ mile in order to arrive just as school begins today?

A 4

B 6

C 8

D 10

E 12

Solution(s):

If George normally walks 1 mile at 3 miles per hours, it will take him $\frac{1}{3}$ hours to walk to school.

Today, he walked $\frac{1}{2}$ miles at 2 miles per hours, which means he walked for

$$\frac{1}{2} \div 2 = \frac{1}{4}$$

hours.

This means he has

$$\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

hours left to run to school. In order to run $\frac{1}{2}$ mile in $\frac{1}{12}$ hours, he must run at a speed of

$$\frac{1}{2} \div \frac{1}{12} = 6$$

miles per hour.

Thus, **B** is the correct answer.

18. Four children were born at City Hospital yesterday. Assume each child is equally likely to be a boy or a girl. Which of the following outcomes is most likely?

A all 4 are boys

B all 4 are girls

C 2 are girls and 2 are boys

D 3 are of one gender and 1 is one of the other gender

E all of these outcomes are equally likely

Solution(s):

We can count the number of ways for each outcome to occur, and whichever has the most is the answer.

A: There is only one way for the children to all be boys.

B: There is only one way for the children to all be girls.

C: This scenario has 6 possibilities:

$$\begin{gathered}\text{BBGG}\\\text{BGBG}\\\text{BGGB}\\\text{GBBG}\\\text{GBGB}\\\text{GGBB}\end{gathered}$$

D: This scenario has 8 possibilities:

$$\begin{gathered}\text{BBBG}\\\text{BBGB}\\\text{BGBB}\\\text{GBBB}\end{gathered}$$

(swapping G and B yields the other 4 possibilities).

E: Clearly, this is not the case.

Thus, **D** is the correct answer.

19. A cube with 3-inch edges is to be constructed from 27 smaller cubes with 1-inch edges. Twenty-one of the cubes are colored red and 6 are colored white.

If the 3-inch cube is constructed to have the smallest possible white surface area showing, what fraction of the surface area is white?

A $\frac{5}{54}$

B $\frac{1}{9}$

C $\frac{5}{27}$

D $\frac{2}{9}$

E $\frac{1}{3}$

Solution(s):

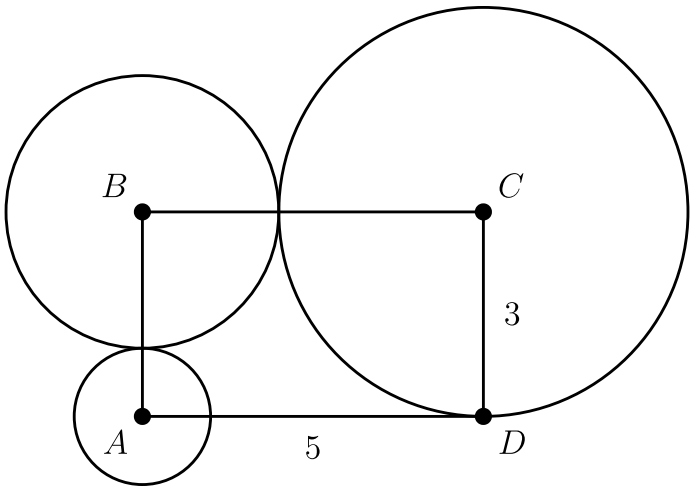
Note that the interior cube has none of its faces showing, which means that this must be a white cube to minimize the white surface area. We should also place the other 5 white cubes in the middle of every space to minimize the number of white faces showing.

This results in a white surface area of 5 in^2. The total surface area is $6\text{ in} \cdot (3\text{ in})^2 = 54\text{ in}^2$.

Therefore, the fraction of white surface area is $\frac{5}{54}$.

Thus, **A** is the correct answer.

20. Rectangle $ABCD$ has sides $CD = 3$ and $DA = 5$. A circle of radius 1 is centered at A, a circle of radius 2 is centered at B, and a circle of radius 3 is centered at C. Which of the following is closest to the area of the region inside the rectangle but outside all three circles?



A 3.5

B 4.0

C 4.5

D 5.0

E 5.5

Solution(s):

The three sectors inside the rectangle are all quarter-circles, with areas $\frac{\pi}{4}$, π, and $\frac{9\pi}{4}$, and therefore their total area is $\frac{7\pi}{2}$.

We can use $\frac{22}{7}$ as an approximation for π. Substituting this value in, we get $\frac{7}{2} \cdot \frac{22}{7} = 11$. The area of the rectangle is $3 \cdot 5 = 15$, so the area outside the circles is (approximately) $15 - 11 = 4$.

Thus, **B** is the correct answer.

21. The 7-digit numbers $\underline{74A52B1}$ and $\underline{326AB4C}$ are each multiples of 3. Which of the following could be the value of C?

A 1

B 2

C 3

D 5

E 8

Solution(s):

For a number to be divisible by 3, the sum of the digits has to be divisible by 3. The sum of the digits of $\underline{74A52B1}$ is $19 + A + B$, so this has to be divisible by 3.

Similarly for $\underline{326AB4C}$, we get that 3 divides $15 + A + B + C$, which means that 3 divides $A + B + C$.

From the first condition, we get that $A + B$ is 1 less than a multiple of 3. This means that C must be 1 more than a multiple of 3.

Therefore, the only way that all these conditions can be satisfied is if $C = 1$.

Thus, **A** is the correct answer.

22. A 2-digit number is such that the product of the digits plus the sum of the digits is equal to the number. What is the units digit of the number?

A 1

B 3

C 5

D 7

E 9

Solution(s):

We can represent the number as $10a + b$, where a and b are both digits. Then the condition implies that

$$ab + a + b = 10a + b.$$

Simplifying this, we get $ab = 9a$, which shows that $b = 9$ since a cannot equal 0.

Thus, **E** is the correct answer.

23. Three members of the Euclid Middle School girls' softball team had the following conversation.

Ashley: I just realized that our uniform numbers are all 2-digit primes.

Brittany : And the sum of your two uniform numbers is the date of my birthday earlier this month.

Caitlin: That's funny. The sum of your two uniform numbers is the date of my birthday later this month.

Ashley: And the sum of your two uniform numbers is today's date.

What number does Caitlin wear?

A 11

B 13

C 17

D 19

E 23

Solution(s):

Since there are no more than 31 days in a month, the sum of any two girls' uniform numbers must be less than or equal to 31. The only possible such pairs of 2-digit primes are

$$11+13=24$$

$$11+17=28,$$

$$11+19=30$$

$$13+17=30.$$

This shows that the desired dates are $24, 28$, and 30. Caitlin's birthday should be the latest, which means that her uniform number must be the smallest.

Therefore, Caitlin's number is 11.

Thus, **A** is the correct answer.

24. One day the Beverage Barn sold 252 cans of soda to 100 customers, and every customer bought at least one can of soda. What is the maximum possible median number of cans of soda bought per customer on that day?

A 2.5

B 3.0

C 3.5

D 4.0

E 4.5

Solution(s):

We can order the number of cans bought by customer from least to greatest. In this ordering, the median is the average of the 50th and 51st customer.

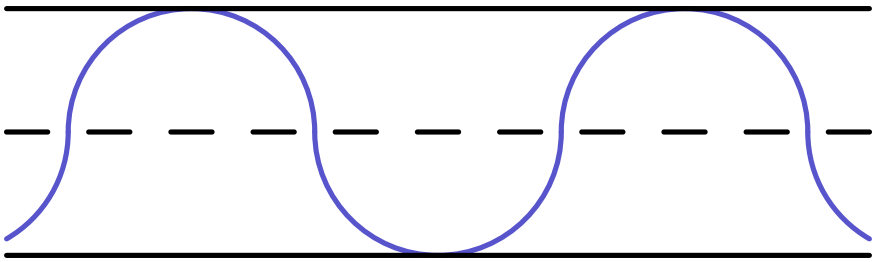
To maximize this, we want to minimize the first 49 purchases. To minimize them, we can set them all equal to 1. If the 50th number is 4, then there would be 51 purchases greater than or equal to 4.

This would cause the total number of cans sold to be at least $49 + 51 \cdot 4 = 253$, which is too large. This forces the 50th purchase to be 3 cans, which lowers the total by 1 to the desired amount. This means that the 51st person bought 4 cans, causing the median to be

$$(3 + 4)/2 = 3.5.$$

Thus, **C** is the correct answer.

25. A straight one-mile stretch of highway, 40 feet wide, is closed. Robert rides his bike on a path composed of semicircles as shown. If he rides at 5 miles per hour, how many hours will it take to cover the one-mile stretch?



Note: 1 mile = 5280 feet

A $\frac{\pi}{11}$

B $\frac{\pi}{10}$

C $\frac{\pi}{5}$

D $\frac{2\pi}{5}$

E $\frac{2\pi}{3}$

Solution(s):

If we compare a straight portion to a semi-circular path, we can see that the semi-circular path is $\frac{\pi}{2}$ longer than the straight path due to the equation for circumference.

The diameter of each semi-circular path is 80 feet, which evenly divides a mile, showing that there are an integer number of semi-circular paths. Therefore, the amount of time it takes Robert to ride along the semi-circular path is directly proportional to the length of the straight path.

The straight path would take Robert $\frac{1}{5}$ hours to ride. This means that it would take him

$$\frac{1}{5} \cdot \frac{\pi}{2} = \frac{\pi}{10}$$

hours to ride along the semi-circular path.

Thus, **B** is the correct answer.

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