2016 AMC 8 Solutions

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https://live.poshenloh.com/past-contests/amc8/2016/solutions

1. The longest professional tennis match ever played lasted a total of 11 hours and 5 minutes. How many minutes was this?

- A 605
- B 655
- C 665
- D 1005
- E 1105

Solution(s):

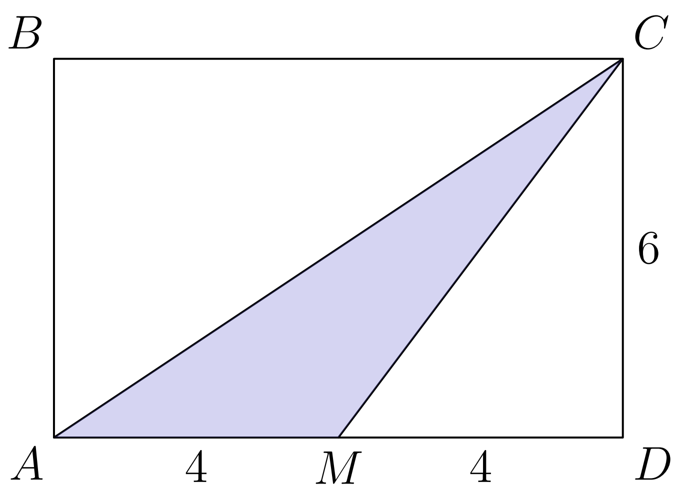
There are 60 minutes in an hour, so the total time is $60 \cdot 11 + 5 = 665$ minutes.

Thus, **C** is the correct answer.

2. In rectangle $ABCD$, $AB = 6$ and $AD = 8$. Point M is the midpoint of $\overline{AD}$. What is the area of $\triangle AMC$?

- **A** 12
- **B** 15
- **C** 18
- **D** 20
- **E** 24

Solution(s):



From the diagram, we can see that the base of $\triangle AMC$ is 4 and the altitude is 4. The area is therefore $\frac{1}{2} \cdot 4 \cdot 6 = 12$.

Thus, **A** is the correct answer.

3. Four students take an exam. Three of their scores are $70, 80$, and 90. If the average of their four scores is 70, then what is the remaining score?

A 40

B 50

C 55

D 60

E 70

Solution(s):

From the average, we can calculate the sum of the scores to be $4 \cdot 70 = 280$. This means that the remaining score is

$$280 - 70 - 80 - 90 = 40.$$

Thus, **A** is the correct answer.

4. When Cheenu was a boy he could run 15 miles in 3 hours and 30 minutes. As an old man he can now walk 10 miles in 4 hours. How many minutes longer does it take for him to walk a mile now compared to when he was a boy?

- A 6
- B 10
- C 15
- D 18
- E 30

Solution(s):

To better compare the rates, we can change his speed into minutes per mile.

As a boy he ran 15 miles in $3 \cdot 60 + 30 = 210$ minutes, which means that he ran at a pace of $210/15 = 14$ minutes per mile.

As an adult, he can walk 10 miles in $4 \cdot 60 = 240$ minutes, which means he walks at a pace of $240/10 = 24$ minutes per mile.

Subtracting the two, we get that he takes 10 more minutes to walk a mile as an adult.

Thus, **B** is the correct answer.

5. The number N is a two-digit number with the following properties:

- When N is divided by 9, the remainder is 1.
- When N is divided by 10, the remainder is 3.

What is the remainder when N is divided by 11?

A 0

B 2

C 4

D 5

E 7

Solution(s):

The two-digit numbers that leave a remainder of 1 when divided by 9 are:

$$\begin{gathered}10, 19, 28, 37, 46, \\ 55, 64, 73, 82, 91.\end{gathered}$$

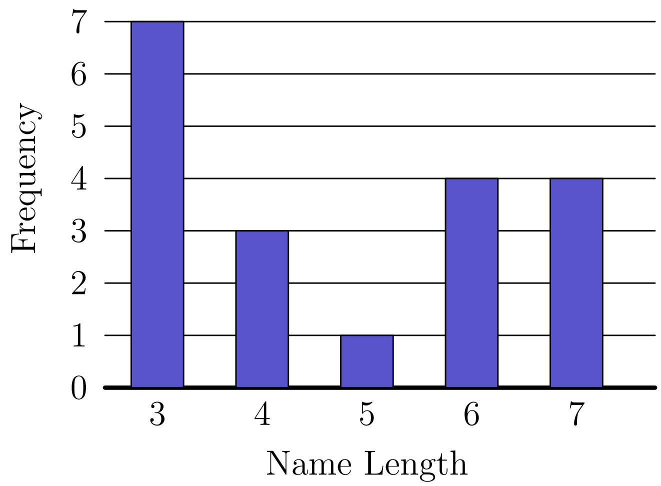
The two-digit numbers that leave a remainder of 3 when divided by 10 are:

$$\begin{gathered}13, 23, 33, 43, 53, \\ 63, 73, 83, 93.\end{gathered}$$

Among these numbers, 73 is the only common number. The remainder of 73 when divided by 11 is 7.

Thus, **E** is the correct answer.

6. The following bar graph represents the length (in letters) of the names of 19 people. What is the median length of these names?



A 3

B 4

C 5

D 6

E 7

Solution(s):

Since there are 19 people, each with one corresponding name length, the middle length will be the tenth one. Counting from the left side, the tenth value that we arrive upon is 4.

Thus, **B** is the correct answer.

7. Which of the following numbers is not a perfect square?

A 1^{2016}

B 2^{2017}

C 3^{2018}

D 4^{2019}

E 5^{2020}

Solution(s):

Since any number with an even exponent is a perfect square, we can eliminate **A**, **C**, and **E**. Also, a square number to any power remains a square number, so that rules out **D**.

Thus, **B** is the correct answer.

8. Find the value of the expression

$$100 - 98 + 96 - 94 + 92 - 90 \\ + \cdots 8 - 6 + 4 - 2.$$

A 20

B 40

C 50

D 80

E 100

Solution(s):

We can group the sum as follows:

$$(100 - 98) + (96 - 94) \\ + \cdots + (4 - 2).$$

Note that each pair evaluates to 2 and there are 25 pairs. Therefore, the total sum is $2 \cdot 25 = 50$.

Thus, **C** is the correct answer.

9. What is the sum of the distinct prime integer divisors of 2016?

A 9

B 12

C 16

D 49

E 63

Solution(s):

We can prime factorize 2016 as $2^5 \cdot 3^2 \cdot 7$. This shows that the prime divisors of 2016 are $2, 3,$ and 7. The sum of these is 12, so **B** is the correct answer.

10. Suppose that $a * b$ means $3a - b$. What is the value of x if

$$2 * (5 * x) = 1?$$

A $\frac{1}{10}$

B 2

C $\frac{10}{3}$

D 10

E 14

Solution(s):

We can simplify the equation as follows:

$$\begin{aligned} 1 &= 2 * (5 * x) \\ &= 2 * (3 \cdot 5 - x) \\ &= 2 * (15 - x) \\ &= 3 \cdot 2 - (15 - x) \\ &= x - 9. \end{aligned}$$

Solving yields $x = 10$.

Thus, **D** is the correct answer.

11. Determine how many two-digit numbers satisfy the following property:

When the number is added to the number obtained by reversing its digits, the sum is 132.

A 5

B 7

C 9

D 11

E 12

Solution(s):

Let ab be the two-digit number in question. Then, it follows that the number obtained by reversing its digits is ba. Therefore, in order for ab to satisfy the property in the question:

$$\begin{aligned} 10(a+b)+a+b &= 132 \\ 11(a+b) &= 132 \\ a+b &= 12. \end{aligned}$$

The only possible solutions $(a, b$ to this equation, where a, b are both one digit, are:

$$(3,9),(4,8),(5,7),(6,6),$$

$$(7,5),(8,4),(9,3).$$

As such, there are 7 solutions.

Thus, **B** is the correct answer.

12. Jefferson Middle School has the same number of boys and girls. Three-fourths of the girls and two-thirds of the boys went on a field trip. What fraction of the students on the field trip were girls?

A $\frac{1}{2}$

B $\frac{9}{17}$

C $\frac{7}{13}$

D $\frac{2}{3}$

E $\frac{14}{15}$

Solution(s):

To more easily compare, we can convert the fractions to have the same denominator:

$$\frac{3}{4} = \frac{9}{12}$$

$$\frac{2}{3} = \frac{8}{12}$$

This shows that the ratio of girls to boys is $9 : 8$, which means that the fraction of girls on the field trip is $\frac{9}{17}$.

Thus, **B** is the correct answer.

13. Two different numbers are randomly selected from the set

$$\{-2, -1, 0, 3, 4, 5\}$$

and multiplied together. What is the probability that the product is 0?

A $\frac{1}{6}$

B $\frac{1}{5}$

C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{1}{2}$

Solution(s):

The only way for the product to be 0 is if one of the number chosen is 0. If the first number chosen is 0, then there are 5 options for the second number.

Similarly, there are 5 combinations if 0 was chosen second.

Therefore, there are 10 total pairs where the product is 0. The total number of pairs is $6 \cdot 5 = 30$, so the probability is

$$\frac{10}{30} = \frac{1}{3}.$$

Thus, **D** is the correct answer.

14. Karl's car uses a gallon of gas every 35 miles, and his gas tank holds 14 gallons when it is full.

One day, Karl started with a full tank of gas, drove 350 miles, bought 8 gallons of gas, and continued driving to his destination. When he arrived, his gas tank was half full. How many miles did Karl drive that day?

- A 525
- B 560
- C 595
- D 665
- E 735

Solution(s):

If Karl drove 350 miles, then he used $350/35$ gallons of gas.

When he bought more gas, he added 8 gallons to $14 - 10 = 4$ gallons, attaining a total of 12 gallons.

If his tank was half full when he arrived, he used $12 - 7 = 5$ gallons, which equates to $5 \cdot 35 = 175$ miles.

Therefore, he travelled a total distance of

$$350 + 175 = 525 \text{ miles.}$$

Thus, **A** is the correct answer.

15. What is the largest power of 2 that is a divisor of $13^4 - 11^4$?

A 8

B 16

C 32

D 64

E 128

Solution(s):

We can factor this expression using difference of squares.

$$\begin{aligned}(13^2 + 11^2)(13^2 - 11^2) \\ &= 290 \cdot 48 \\ &= 32 \cdot 145 \cdot 3\end{aligned}$$

This shows that 32 is the largest power of 2 that divides the expression.

Thus, **C** is the correct answer.

16. Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead, because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie?

A $1\frac{1}{4}$

B $3\frac{1}{3}$

C 4

D 5

E 25

Solution(s):

Since Annie is 25% faster than Bonnie, for every lap Bonnie finishes, Annie completes $1\dfrac{1}{4}$ laps. Therefore, Annie gains a quarter lap every time Bonnie finished a lap.

With this in mind, for Annie to completely lap Bonnie, Bonnie must finish 4 laps, which means that Annie finished 5 laps.

Thus, **D** is the correct answer.

17. An ATM password at Fred's Bank is composed of four digits from 0 to 9, with repeated digits allowable. If no password may begin with the sequence $9, 1, 1$, then how many passwords are possible?

- A 30
- B 7290
- C 9000
- D 9990
- E 9999

Solution(s):

The total number of passwords with no conditions is 10^4. The condition removes 10 possible passwords since the first 3 are determined, and the last one can be anything. Therefore, the number of acceptable passwords is

$$10,000 - 10 = 9990.$$

Thus, **D** is the correct answer.

18. In an All-Area track meet, 216 sprinters enter a $100-$meter dash competition. The track has 6 lanes, so only 6 sprinters can compete at a time. At the end of each race, the five non-winners are eliminated, and the winner will compete again in a later race.

How many races are needed to determine the champion sprinter?

A 36

B 42

C 43

D 60

E 72

Solution(s):

Note that each race eliminates 5 people. For there to be a winner, 215 must be eliminated. Therefore, $215/15 = 43$ races are required to eliminate this number of people.

Thus, **C** is the correct answer.

19. The sum of 25 consecutive even integers is $10,000$. What is the largest of these 25 consecutive integers?

A 360

B 388

C 412

D 416

E 424

Solution(s):

The average of these numbers is $10,000/25 = 400$. The largest number is 12 even numbers away, which means that it equals $400 + 12 \cdot 2 = 424$.

Thus, **E** is the correct answer.

20. The least common multiple of a and b is 12, and the least common multiple of b and c is 15. What is the least possible value of the least common multiple of a and c?

- A 20
- B 30
- C 60
- D 120
- E 180

Solution(s):

We know that b has to divide both 12 and 15, so it must equal either 1 or 3.

If $b = 1$, then $a = 12$ and $c = 15$, making their least common multiple 60. If $b = 3$, then the smallest value of a is 12 and c is 5. The least common multiple in this scenario is 20.

Thus, **A** is the correct answer.

21. A top hat contains 3 red chips and 2 green chips. Chips are drawn randomly, one at a time without replacement, until all 3 of the reds are drawn or until both green chips are drawn. What is the probability that the 3 reds are drawn?

A $\frac{3}{10}$

B $\frac{2}{5}$

C $\frac{1}{2}$

D $\frac{3}{5}$

E $\frac{2}{3}$

Solution(s):

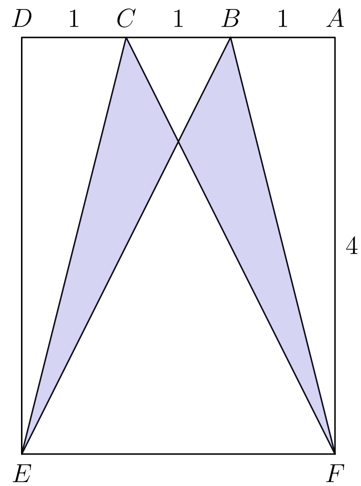
The only way for the 3 reds to be drawn first is if there is only one green drawn in the first 4 draws. The green can be in any of the first 4 spots, yielding 4 possibilities.

We can find the total number of possibilities to be 10 by listing them out. Therefore, the desired probability is

$$\frac{4}{10} = \frac{2}{5}.$$

Thus, **B** is the correct answer.

22. Rectangle $DEFA$ below is a 3×4 rectangle with $DC = CB = BA = 1$. The area of the "bat wings" (shaded area) is



A 2

B $2\frac{1}{2}$

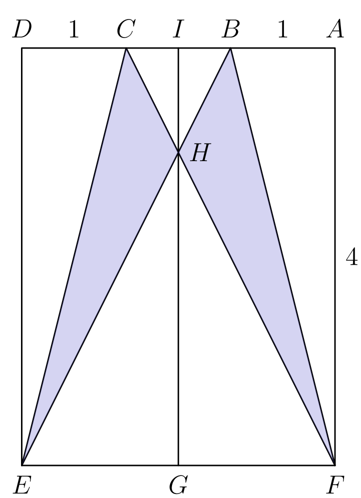
C 3

D $3\frac{1}{2}$

E 5

Solution(s):

Define I to be the midpoint of $\overline{AD}$ and G to be the midpoint of $\overline{EF}$. Also define H to be the intersection of $\overline{CF}$ and $\overline{BE}$.



The area of $\triangle BCE$ equals

$$\frac{1}{2} \cdot 1 \cdot 4 = 2.$$

By symmetry, we can see that $\triangle BCH$ and $\triangle EFH$ are similar. Since their bases are in a $1:3$ ratio, so are their altitudes. This means that $3IH = HG$, which implies that $IH = 1$.

Therefore, the area of

$$\begin{aligned} \triangle BCH &= \frac{1}{2} \cdot 1 \cdot 1 \\ &= \frac{1}{2}. \end{aligned}$$

This implies that the area of

$$\begin{aligned} \triangle ECH &= 2 - \frac{1}{2} \\ &= \frac{3}{2}. \end{aligned}$$

Since the figure is symmetric, the total area of the bat wings is $2 \cdot \dfrac{3}{2} = 3$.

Thus, **C** is the correct answer.

23. Two congruent circles centered at points A and B each pass through the other circle's center. The line containing both A and B is extended to intersect the circles at points C and D.

The circles intersect at two points, one of which is E. What is the degree measure of $\angle CED$?

A 90

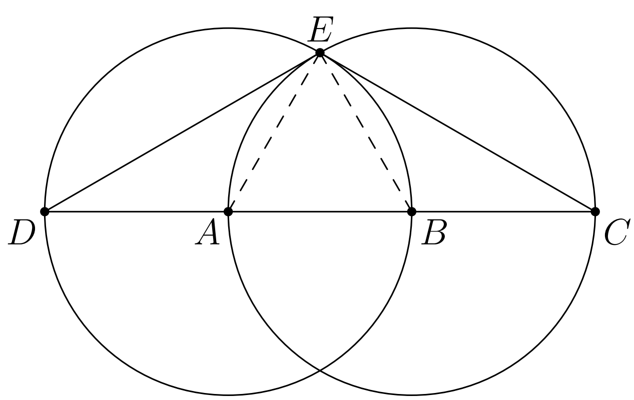
B 105

C 120

D 135

E 150

Solution(s):



We know that $AE = EB = AB$ since they are all radii of congruent circles, so they form an equilateral triangle, which means that $\angle AEB = 60^\circ$.

Also, since $\overline{DB}$ and $\overline{AC}$ are diameters,

$$\angle DEB = \angle AEC = 90^\circ.$$

Therefore,

$$\angle CED = \angle DEB + \angle AEC - \angle AEB,$$

which equals 120°.

Thus, **C** is the correct answer.

24. The digits $1, 2, 3, 4$, and 5 are each used once to write a five-digit number $PQRST$. The three-digit number PQR is divisible by 4, the three-digit number QRS is divisible by 5, and the three-digit number RST is divisible by 3. What is P?

A 1

B 2

C 3

D 4

E 5

Solution(s):

Since QRS is divisible by 5, we know that $S = 5$.

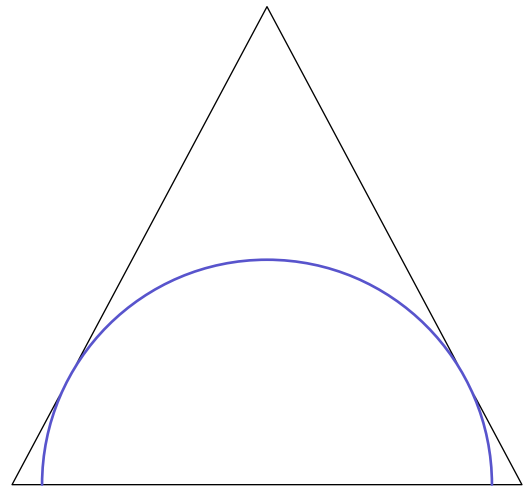
Since PQR is divisible by 4, QR equals either $12, 24$, or 32.

This means that RST will equal either $25T$ or $45T$. Note that $RST = 25T$ would require T to be 2 or 5, each of which has solutions. Therefore the remaining digits when considering $RST = 45T$, 453 is the only number divisible by \(3.\

Therefore, $PQRST = 12435$.

Thus, **A** is the correct answer.

25. A semicircle is inscribed in an isosceles triangle with base 16 and height 15 so that the diameter of the semicircle is contained in the base of the triangle as shown. What is the radius of the semicircle?



A $4\sqrt{3}$

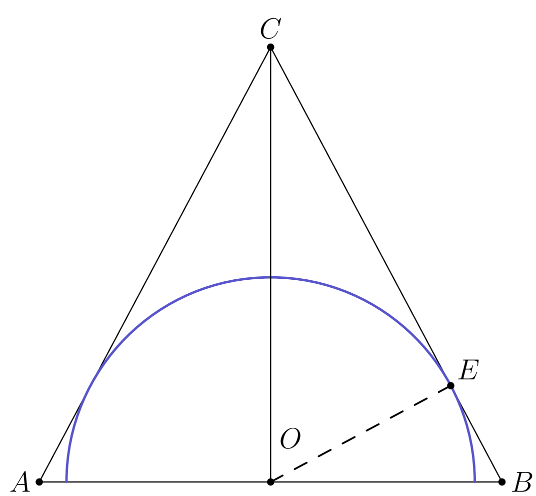
B $\frac{120}{17}$

C 10

D $\frac{17\sqrt{2}}{2}$

E $\frac{17\sqrt{3}}{2}$

Solution(s):



Let O be the center of the circle, which is the midpoint of $\overline{AB}$.

We then get that $BC = 17$ via the Pythagorean theorem.

In addition, we can also calculate the area of $\triangle BOC$ as:

$$\begin{aligned}\triangle BOC &= \frac{1}{2} \cdot 8 \cdot 15 \\ &= 60.\end{aligned}$$

As the area of $\triangle BOC = 60 = \dfrac{1}{2} OE \cdot CB$ we can see that

$$OE = \frac{120}{17}.$$

Thus, **B** is the correct answer.

Problems: https://live.poshenloh.com/past-contests/amc8/2016