2019 AMC 8 Solutions

Typeset by: LIVE, by Po-Shen Loh
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1. Ike and Mike go into a sandwich shop with a total of $30.00 to spend. Sandwiches cost $4.50 each and soft drinks cost $1.00 each. Ike and Mike plan to buy as many sandwiches as they can, and use any remaining money to buy soft drinks. Counting both sandwiches and soft drinks, how many items will they buy?

A 6

B 7

C 8

D 9

E 10

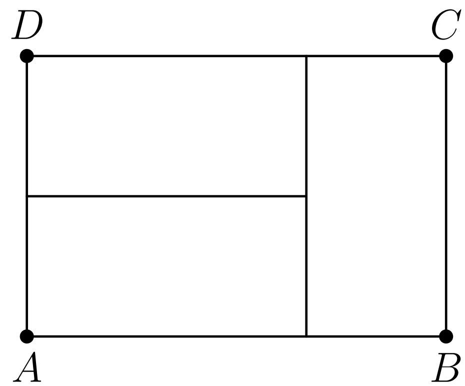
Solution(s):

If they buy 6 sandwiches, they would spend $6 \times 4.5 = 27$ dollars. This means that they would not be able to buy any more sandwiches.

They would then have $30 - 27 = 3$ dollars left. With this, they could buy 3 sodas. They would therefore buy a total of $6 + 3 = 9$ items.

Thus, the correct answer is **D**.

2. Three identical rectangles are put together to form rectangle $ABCD$, as shown in the figure below. Given that the length of the shorter side of each of the smaller rectangles is 5 feet, what is the area in square feet of rectangle $ABCD$?



A 45

B 75

C 100

D 125

E 150

Solution(s):

From the figure, we can see that the longer side has the same length as two of the shorter sides. This makes it $2 \cdot 5 = 10$ feet long.

This tells us that $BC = 10$ feet and $DC = 10 + 5 = 15$ feet. Therefore, the area would be $10 \cdot 15 = 150 \text{ ft.}^2$

Thus, the correct answer is **E**.

3. Which of the following is the correct order of the fractions $\frac{15}{11}, \frac{19}{15}$, and $\frac{17}{13}$, from least to greatest?

A $\frac{15}{11} < \frac{17}{13} < \frac{19}{15}$

B $\frac{15}{11} < \frac{19}{15} < \frac{17}{13}$

C $\frac{17}{13} < \frac{19}{15} < \frac{15}{11}$

D $\frac{19}{15} < \frac{15}{11} < \frac{17}{13}$

E $\frac{19}{15} < \frac{17}{13} < \frac{15}{11}$

Solution(s):

We can rewrite the fraction as follows:

$$\frac{15}{11} = 1 + \frac{4}{11}$$
$$\frac{17}{13} = 1 + \frac{4}{13}$$
$$\frac{19}{15} = 1 + \frac{4}{15}.$$

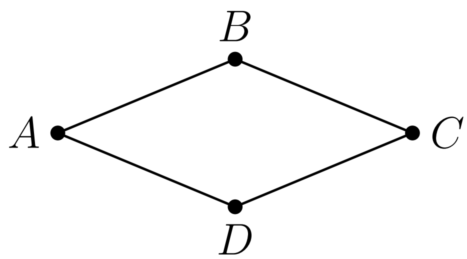
Recall that if two fractions have the same numerator, then the fraction with the larger denominator is smaller.

Using this fact, we can see that the correct ordering is

$$\frac{19}{15} < \frac{17}{13} < \frac{15}{11}.$$

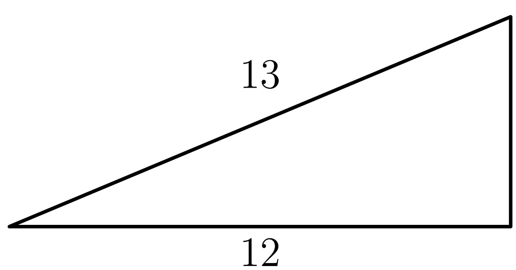
Thus, the correct answer is **E**.

4. Quadrilateral $ABCD$ is a rhombus with perimeter 52 meters. The length of diagonal $\overline{AC}$ is 24 meters. What is the area in square meters of rhombus $ABCD$?



- A 60
- B 90
- C 105
- D 120
- E 144

Solution(s):



We can split the rhombus up into 4 triangles, each of which looks like this.

We know that this is a right triangle, since the diagonals of a rhombus are perpendicular, and we know the hypotenuse is $52 \div 4 = 13$.

Using the Pythagorean theorem, we get the other leg to be

$$\sqrt{13^2 - 12^2} = \sqrt{25} = 5.$$

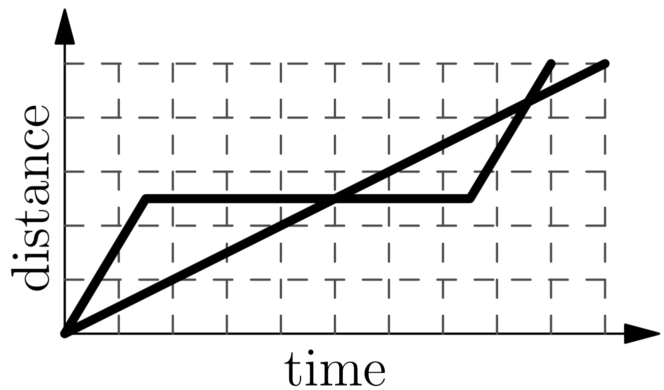
This means that the area of the rhombus is

$$4 \cdot \frac{5 \cdot 12}{2} = 4 \cdot 30 = 120.$$

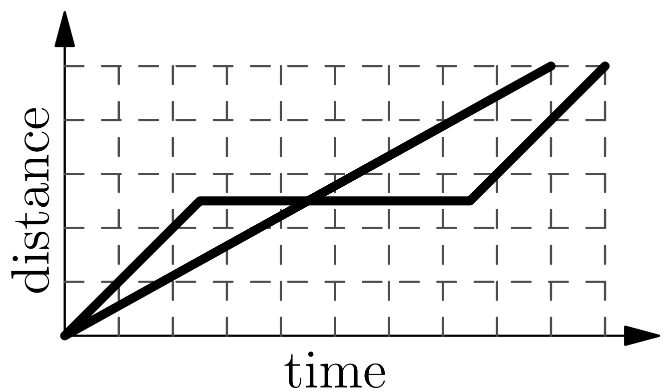
Thus, the correct answer is **D**.

5. A tortoise challenges a hare to a race. The hare eagerly agrees and quickly runs ahead, leaving the slow-moving tortoise behind. Confident that he will win, the hare stops to take a nap. Meanwhile, the tortoise walks at a slow steady pace for the entire race. The hare awakes and runs to the finish line, only to find the tortoise already there. Which of the following graphs matches the description of the race, showing the distance d traveled by the two animals over time t from start to finish?

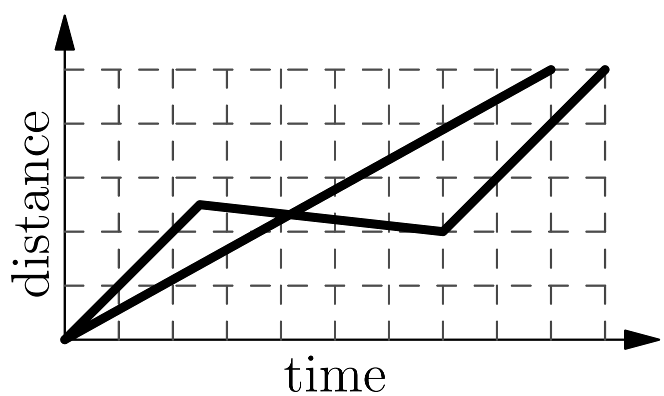
A



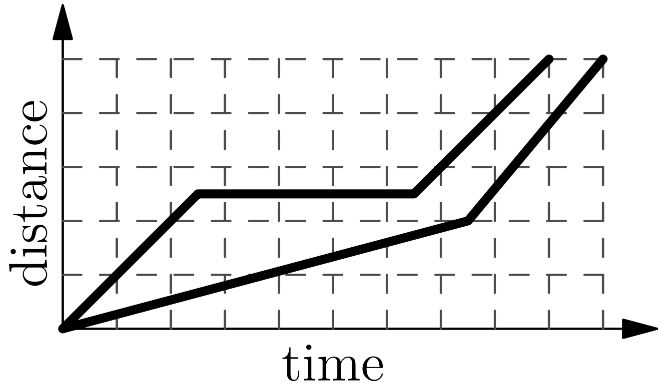
B



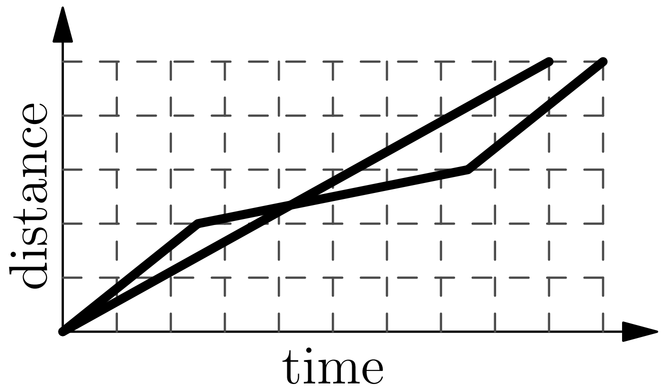
C



D



E



Solution(s):

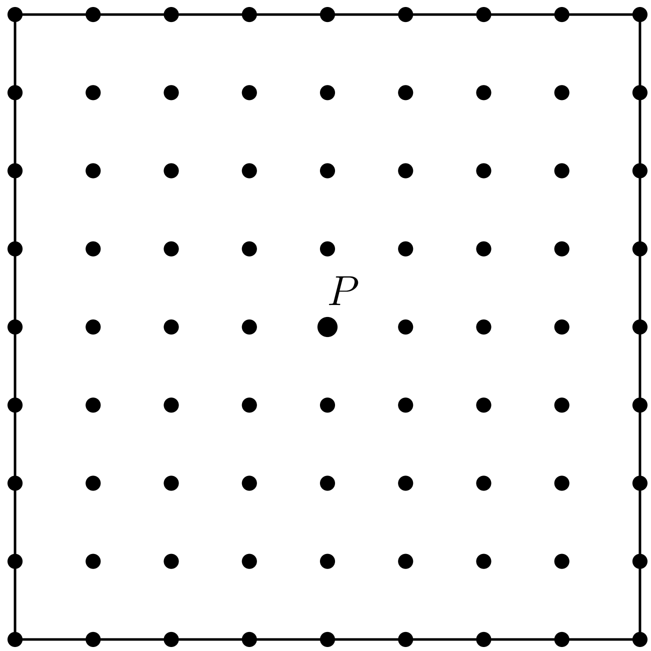
The hare's path is the one with the horizontal line in the middle, which represent the period where the hare took a nap.

We also know that the time it takes the hare to reach the finish line is longer than the time it took the tortoise.

This means that the end of the hare's path should have a larger x-value. This is exactly graph B.

Thus, the correct answer is **B**.

6. There are 81 grid points (uniformly spaced) in the square shown in the diagram below, including the points on the edges. Point P is in the center of the square. Given that point Q is randomly chosen among the other 80 points, what is the probability that the line PQ is a line of symmetry for the square?



A $\frac{1}{5}$

B $\frac{1}{4}$

C $\frac{2}{5}$

D $\frac{9}{20}$

E $\frac{1}{2}$

Solution(s):

Note the only lines of symmetry for a square are the two diagonals and the two lines connecting opposite midpoints.

Therefore, Q must be a point on any one of these four lines. Each line consists of 9 points, for a total of $4 \cdot 9 = 36$ points.

Note that Q cannot be the same as P, so we have to subtract out the 4 points which coincide with P, for a total of $36 - 4 = 32$ points.

The probability is therefore $\dfrac{32}{80} = \dfrac{2}{5}$.

Thus, the correct answer is **C**.

7. Shauna takes five tests, each worth a maximum of 100 points. Her scores on the first three tests are $76, 94,$ and 87. In order to average 81 for all five tests, what is the lowest score she could earn on one of the other two tests?

A 48

B 52

C 66

D 70

E 74

Solution(s):

To minimize one of the scores, we have to maximize the other score. Assume that Shauna gets a 100 on her fourth test.

The sum of the 4 tests is then

$$76 + 94 + 87 + 100 = 357.$$

For an average of 81, Shauna's test scores must add to $5 \cdot 81 = 405$. This means that she needs to get a $405 - 357 = 48$ on her last test.

Thus, the correct answer is **A**.

8. Gilda has a bag of marbles. She gives 20% of them to her friend Pedro. Then Gilda gives 10% of what is left to another friend, Ebony. Finally, Gilda gives 25% of what is now left in the bag to her brother Jimmy. What percentage of her original bag of marbles does Gilda have left for herself?

A 20

B $33\frac{1}{3}$

C 38

D 45

E 54

Solution(s):

Assume that Gilda starts off with 100 marbles. After giving 20 marbles to Pedro, she has $100 - 20 = 80$ marbles left.

She then gives $80 \times .1 = 8$ marbles to Ebony, with her having $80 - 8 = 72$ marbles left.

Finally, Gilda gives $72 \times .25 = 18$ marbles to Jimmy. She has a total of $72 - 18 = 54$ marbles left, which is 54% of her original total.

Thus, the correct answer is **E**.

9. Alex and Felicia each have cats as pets. Alex buys cat food in cylindrical cans that are 6 cm in diameter and 12 cm high. Felicia buys cat food in cylindrical cans that are 12 cm in diameter and 6 cm high. What is the ratio of the volume of one of Alex's cans to the volume of one of Felicia's cans?

A $1:4$

B $1:2$

C $1:1$

D $2:1$

E $4:1$

Solution(s):

Felicia's diameter is twice as much as Alex's which means that her radius is also twice as much.

Recall that the formula for the area of a circle is πr^2. If the radius is doubled, then the area the is quadrupled.

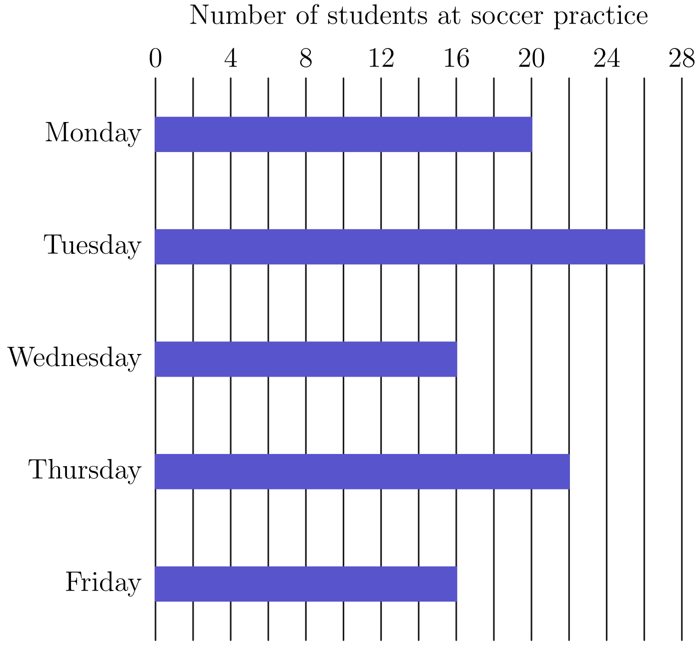
This means that the base of Felicia's can has 4 times the area as the base of Alex's can.

We also see that Alex's can is twice as tall as Felicia's can. The formula for the volume of a can is base times height.

Using this formula, we see that Felicia's can will have twice the volume of Alex's can since $4 \div 2 = 2$.

Thus, the correct answer is **B**.

10. The diagram shows the number of students at soccer practice each weekday during last week. After computing the mean and median values, Coach discovers that there were actually 21 participants on Wednesday. Which of the following statements describes the change in the mean and median after the correction is made?



A The mean increases by 1 and the median does not change.

B The mean increases by 1 and the median increases by 1.

C The mean increases by 1 and the median increases by 5.

D The mean increases by 5 and the median increases by 1.

E The mean increases by 5 and the median increases by 5.

Solution(s):

There are 5 more participants, which means the mean is increased by $5 \div 5 = 1$.

Right now, we can see that median 20. If 16 gets replaced with 21, then the median becomes 21.

This shows that the median is also increased by 1.

Thus, the correct answer is **B**.

11. The eighth grade class at Lincoln Middle School has 93 students. Each student takes a math class or a foreign language class or both. There are 70 eighth graders taking a math class, and there are 54 eight graders taking a foreign language class. How many eight graders take *only* a math class and *not* a foreign language class?

A 16

B 23

C 31

D 39

E 70

Solution(s):

The number of kids that are taking both classes is

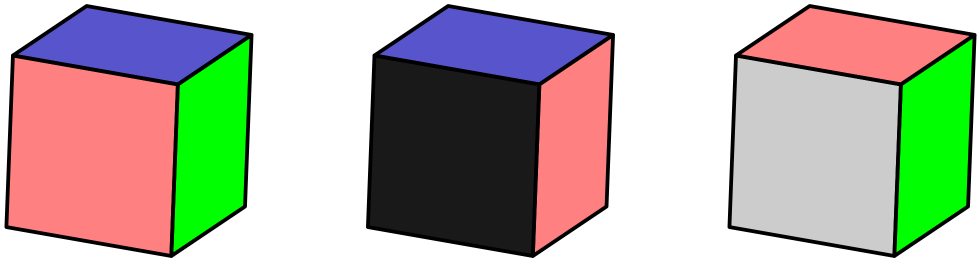
$$70 + 54 - 93 = 31.$$

Subtracting this from the number of kids taking a math class will give us the number of kids taking only a math class.

The desired number is $70 - 31 = 39.$

Thus, the correct answer is **D**.

12. The faces of a cube are painted in six different colors: red, white, green, purple, yellow, and black. Three views of the cube are shown below. What is the color of the face opposite the yellow face?



A red

B white

C green

D purple

E black

Solution(s):

Using the first and third cubes, we can see that black is opposite purple (both have red and green connected).

Similarly, with the first and second cubes, we see that green and white are opposites.

This means that yellow must be opposite red.

Thus, the correct answer is **A**.

13. A *palindrome* is a number that has the same value when read from left to right or from right to left. (For example, 12321 is a palindrome.) Let N be the least three-digit integer which is not a palindrome but which is the sum of three distinct two-digit palindromes. What is the sum of the digits of N?

A 2

B 3

C 4

D 5

E 6

Solution(s):

Note that 2-digit palindromes have the tens and units digits the same. This means that they are all multiples of 11.

If a 3-digit number is the sum of 3 2-digit palindromes, then it itself must also be a multiple of 11.

The smallest 3-digit multiple of 11 that is not a palindrome is 110. This can be achieved by adding $11 + 22 + 77$.

Therefore, $N = 110$. The sum of its digits is $1 + 1 + 0 = 2$.

Thus, the correct answer is **A**.

14. Isabella has 6 coupons that can be redeemed for free ice cream cones at Pete's Sweet Treats. In order to make the coupons last, she decides that she will redeem one every 10 days until she has used them all. She knows that Pete's is closed on Sundays, but as she circles the 6 dates on her calendar, she realizes that no circled date falls on a Sunday. On what day of the week does Isabella redeem her first coupon?

A. Monday

B. Tuesday

C. Wednesday

D. Thursday

E. Friday

Solution(s):

Let us go through the answer choices.

If we she starts on Monday, her next day is Thursday, and the day after is Sunday.

If we start with Tuesday, the next day is Friday. The day after is Monday, which as we now from above will hit Sunday in 2 more days.

If we start with Wednesday, the next day is Saturday. The day after is Tuesday, which we know hits Sunday only after 4 more iterations.

Therefore, if we start with Wednesday, we loop through 6 iterations with hitting Sunday.

Thus, the correct answer is **C**.

15. On a beach 50 people are wearing sunglasses and 35 people are wearing caps. Some people are wearing both sunglasses and caps. If one of the people wearing a cap is selected at random, the probability that this person is also wearing sunglasses is $\frac{2}{5}$. If instead, someone wearing sunglasses is selected at random, what is the probability that this person is also wearing a cap?

A $\frac{14}{85}$

B $\frac{7}{25}$

C $\frac{2}{5}$

D $\frac{4}{7}$

E $\frac{7}{10}$

Solution(s):

If the probability is $\frac{2}{5}$, that means $\frac{2}{5}$ of the people wearing caps are also wearing sunglasses.

This means that $35 \cdot \frac{2}{5} = 14$ people are wearing both caps and sunglasses.

The probability of a person wearing sunglasses also wearing a cap is $\frac{14}{50} = \frac{7}{25}$.

Thus, the correct answer is **B**.

16. Qiang drives 15 miles at an average speed of 30 miles per hour. How many additional miles will he have to drive at 55 miles per hour to average 50 miles per hour for the entire trip?

A 45

B 62

C 90

D 110

E 135

Solution(s):

For the first 15 miles, Qiang drove for $\frac{15}{30} = \frac{1}{2}$ an hour.

Let x be the distance that Qiang must drive for to satisfy the condition. It will take him $\frac{x}{55}$ hours to drive this distance.

The total trip is now $15 + x$ miles. We know the average speed is 50 miles per hour, so this will take him $\frac{x+15}{50}$ hours.

Setting the two times equal to each other, we get

$$\frac{1}{2} + \frac{x}{55} = \frac{x+15}{50}.$$

Cross-multiplying and simplifying yields

$$15 + x = 25 + \frac{10x}{11} \Rightarrow x = 110.$$

Thus, the correct answer is **D**.

17. What is the value of the product below?

$$\left(\frac{1\cdot 3}{2\cdot 2}\right)\left(\frac{2\cdot 4}{3\cdot 3}\right)\left(\frac{3\cdot 5}{4\cdot 4}\right)\cdots\left(\frac{97\cdot 99}{98\cdot 98}\right)\left(\frac{98\cdot 100}{99\cdot 99}\right)$$

A $\frac{1}{2}$

B $\frac{50}{99}$

C $\frac{9800}{9801}$

D $\frac{100}{99}$

E 50

Solution(s):

We can regroup all the number as follows.

$$\frac{1}{2}\cdot\left(\frac{3\cdot 2}{2\cdot 3}\right)\left(\frac{4\cdot 3}{3\cdot 4}\right)\cdots\left(\frac{99\cdot 98}{98\cdot 99}\right)\cdot\frac{100}{99}$$

From this representation, we can see that all the middle terms cancel leaving only

$$\frac{1}{2}\cdot\frac{100}{99}=\frac{50}{99}.$$

Thus, the correct answer is **B**.

18. The faces of each of two fair dice are numbered $1, 2, 3, 5, 7$, and 8. When the two dice are tossed, what is the probability that their sum will be an even number?

A $\frac{4}{9}$

B $\frac{1}{2}$

C $\frac{5}{9}$

D $\frac{3}{5}$

E $\frac{2}{3}$

Solution(s):

The only two ways that the sum can be even is if both rolls are even or both are odd.

The probability that a roll is even is $\frac{2}{6} = \frac{1}{3}$, and the probability that it is odd is $\frac{4}{6} = \frac{2}{3}$.

Therefore, the desired probability is

$$\frac{1}{3} \cdot \frac{1}{3} + \frac{2}{3} \cdot \frac{2}{3} = \frac{5}{9}.$$

Thus, the correct answer is **C**.

19. In a tournament there are six teams that play each other twice. A team earns 3 points for a win, 1 point for a draw, and 0 points for a loss. After all the games have been played it turns out that the top three teams earned the same number of total points. What is the greatest possible number of total points for each of the top three teams?

A 22

B 23

C 24

D 26

E 30

Solution(s):

We can assume that the top 3 teams won every game against every team not amongst themselves.

They play $3 \cdot 2 = 6$ games in total, getting a total of $6 \cdot 3 = 18$ points.

Now, amongst the top $3,$ each team plays each other time twice. To even out the scores, we can let one team win one game and let the other team win the other game.

This means that for every pair of the top 3 teams, each team in the pair gets 3 points. There are 3 such pairs, with each team appearing in 2 pairs.

This means that each team will get an extra $3 \cdot 2 = 6$ points. Therefore, their maximum score is $18 + 6 = 24.$

Thus, the correct answer is **C**.

20. How many different real numbers x satisfy the equation

$$(x^2-5)^2=16?$$

A 0

B 1

C 2

D 4

E 8

Solution(s):

Recall that

$$a^2-b^2=(a+b)(a-b).$$

We can do the following rearrangement.

$$\begin{gathered}(x^2-5)^2-4^2=0\\(x^2-5+4)(x^2-5-4)=0\\(x+1)(x-1)(x+3)(x-3)=0\end{gathered}$$

From this we can see that there are 4 possible values for x.

Thus, the correct answer is **D**.

21. What is the area of the triangle formed by the lines $y=5$, $y=1+x$, and $y=1-x$?

A 4

B 8

C 10

D 12

E 16

Solution(s):

First, let us find the vertices of the triangles. The intersections between $y=5$ and the other two lines are

$$(4,5) \text{ and } (-4,5).$$

To find the third intersection, we can equate the two equations to get

$$1+x=1-x \Rightarrow x=0.$$

This gives us the point $(0,1)$.

Note that the first two intersection points are mirrored across the y-axis. This tells us that the triangle is isosceles.

The base is therefore $2 \cdot 4=8$, and the height is $5-1=4$. The area of the triangle is therefore

$$\frac{1}{2} \cdot 8 \cdot 4=16.$$

Thus, the correct answer is **E**.

22. A store increased the original price of a shirt by a certain percent and then decreased the new price by the same amount. Given that the resulting price was 84% of the original price, by what percent was the price increased and decreased?

A 16

B 20

C 28

D 36

E 40

Solution(s):

Let x be the percent converted to a real number. Then the percent increase would multiply the original by $1 + x$.

The percent decrease would multiply the new price by $1 - x$. The final price will the be the original price multiplied by

$$(1 + x)(1 - x) = 1 - x^2 = .84.$$

This tells us that

$$x^2 = .16 \Rightarrow x = .4.$$

This tells us that the percent would be 40%.

Thus, the correct answer is **E**.

23. After Euclid High School's last basketball game, it was determined that $\frac{1}{4}$ of the team's points were scored by Alexa and $\frac{2}{7}$ were scored by Brittany. Chelsea scored 15 points. None of the other 7 team members scored more than 2 points. What was the total number of points scored by the other 7 team members?

A 10

B 11

C 12

D 13

E 14

Solution(s):

Let x be the total number of points and y be the desired answer.

Then from the problem statement we get that

$$\frac{x}{4} + \frac{2x}{7} + 15 + y = x.$$

Simplifying yields

$$y + 15 = \frac{13x}{28}.$$

We know that $y \leq 14$ since none of the 7 team members scored more than 2 points.

We also know that x must be a multiple of 28. If $x = 28$, then we get that

$$y + 15 = 13 \Rightarrow y = -2,$$

which is not allowed.

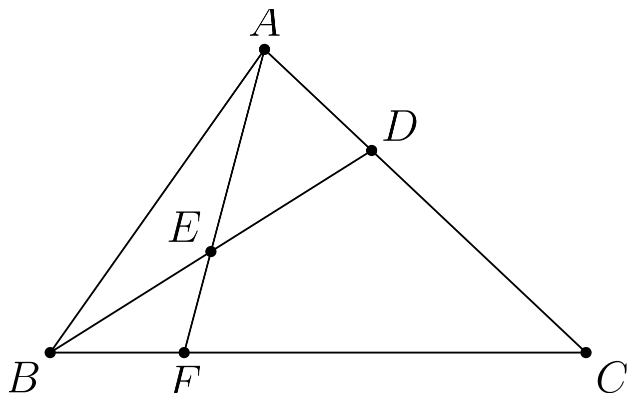
If $x = 2 \cdot 28 = 56$, then we have that

$$y + 15 = 26 \Rightarrow y = 11,$$

which works.

Thus, the correct answer is **B**.

24. In triangle ABC, point D divides side $\overline{AC}$ so that $AD : DC = 1 : 2$. Let E be the midpoint of $\overline{BD}$ and let F be the point of intersection of line BC and line AE. Given that the area of $\triangle ABC$ is 360, what is the area of $\triangle EBF$?



A 24

B 30

C 32

D 36

E 40

Solution(s):

We know that the area of $\triangle BCD$ is twice that of $\triangle ABD$ since its base is twice as long and they have the same heights.

This tells us that the area of $\triangle ABD$ is $360 \div 3 = 120$. Similarly, we get that the area of $\triangle ABE$ is $120 \div 2 = 60$.

Note that $CD = \dfrac{2}{3}CA$. This tells us that the altitude of $\triangle BCD$ is $\dfrac{2}{3}$ the altitude of $\triangle ABC$.

We also know that $BD = \dfrac{1}{2}BD$, which tells us that the altitude of $\triangle BEF$ is $\dfrac{1}{2}$ the altitude of $\triangle BCD$.

Finally, we get that the altitude of $\triangle BEF$ is $\dfrac{2}{3} \cdot \dfrac{1}{2} = \dfrac{1}{3}$ the altitude of $\triangle ABC$.

Note that the altitude of $\triangle ABF$ is the same as the altitude of $\triangle ABC$.

This tells us that the area of $\triangle BEF$ is $\dfrac{1}{3}$ the area of $\triangle ABF$ since they have the same base but different altitudes.

This gives us the following equation, where x equals the area of $\triangle BEF$.

$$\frac{x}{x+60}=\frac{1}{3}.$$

Cross-multiplying yields

$$3x=x+60 \Rightarrow x=30.$$

Thus, the correct answer is **B**.

25. Alice has 24 apples. In how many ways can she share them with Becky and Chris so that each of the three people has at least two apples?

A 105

B 114

C 190

D 210

E 380

Solution(s):

Let us assign everybody 2 apples. This leaves us with $24 - 3 \cdot 2 = 18$ apples.

Then, we want to solve

$$a + b + c = 18,$$

where a is the number of apples that Alice gets and so on.

Note that these are all nonnegative, since we already satisfied the only condition we needed to.

We can use stars and bars to get the number of solutions as

$$\binom{18+3-1}{3-1} = \binom{20}{2} = 190.$$

Thus, the correct answer is **C**.

Problems: https://live.poshenloh.com/past-contests/amc8/2019