# 2020 AMC 8 Solutions

Typeset by: LIVE, by Po-Shen Loh
https://live.poshenloh.com/past-contests/amc8/2020/solutions



**1.** Luka is making lemonade to sell at a school fundraiser. His recipe requires $4$ times as much water as sugar and twice as much sugar as lemon juice. He uses $3$ cups of lemon juice. How many cups of water does he need?

A $6$

B $8$

C $12$

D $18$

E $24$

## Solution(s):

Since Luka needs twice as much sugar as lemon, he needs $2 \cdot 3 = 6$ cups of sugar. Since Luka also needs $4$ times as much water as sugar, he needs $4 \cdot 6 = 24$ cups of water.

Thus, the correct answer is **E**.

**2.** Four friends do yardwork for their neighbors over the weekend, earning \$15, \$20, \$25, and \$40 respectively. They decide to split their earnings equally among themselves. In total how many dollars will the friend who earned \$40 give to the others?

- A $5$
- B $10$
- C $15$
- D $20$
- E $25$

## Solution(s):

First, the total amount of money that they make is \$15+\$20+\$25+\$40=\$100.

Since they divide this equally, they each get $\frac{100}{4} = 25$ dollars.

This means that the person who earned \$40 earned \$15 more than what he will end up with, so he gives \$15 to the others.

Thus, the correct answer is **C**.

**3.** Carrie has a rectangular garden that measures $6$ feet by $8$ feet. She plants the entire garden with strawberry plants. Carrie is able to plant $4$ strawberry plants per square foot, and she harvests an average of $10$ strawberries per plant. How many strawberries can she expect to harvest?

A $560$

B $960$

C $1120$

D $1920$

E $3840$

## Solution(s):

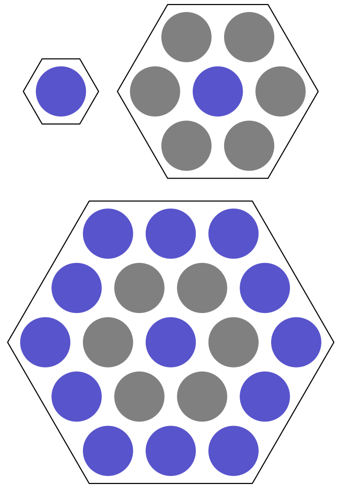
First, the size of the garden is $6\text{ft} \cdot 8\text{ft} = 48\text{ft}^2$.

Next, since there are $4$ plants per square foot, Carrie can plant $4 \cdot 48 = 192$ plants total.

Finally, since there are $10$ strawberries per plant, Carrie can harvest $10 \cdot 192 = 1920$ strawberries total.

Thus, the correct answer is **D**.

**4.** Three hexagons of increasing size are shown below. Suppose the dot pattern continues so that each successive hexagon contains one more band of dots. How many dots are in the next hexagon?



A $35$

B $37$

C $39$

D $43$

E $49$

## Solution(s):

Firstly, let's try and find a pattern for the number of dots in the $n$th ring. Notice that for the $n$th ring, there is always $n$ dots on each edge of the hexagon. However, this overcounts as each vertex of the hexagon will be counted twice.

Therefore, we can claim that for the $n$th ring, the number of dots is equal to $6n - 6 = 6(n-1)$. Note that this pattern does not hold for $n = 1$.

Therefore, for the first hexagon, we have $1$ dot, the 2nd hexagon adds $6(2-1) = 6$ dots, the 3rd hexagon adds $6(3-1) = 12$ dots, as reflected in the diagram. Extending the pattern, we can say that the fourth hexagon adds $6(4-1) = 18$ dots.

Finally, notice that the total number of dots in the hexagon is equal to the sum of all the rings up to the $n$th ring. This is the same as saying:

$$1 + \sum_{i=2}^{n} 6(i-1)$$

Therefore, for $n = 4$ we have a total of

$$\begin{aligned} & 1 + \sum_{i=2}^{4} 6(n-1) \\ & = 1 + 6 + 12 + 18 \\ & = 37 \end{aligned}$$

Thus, the correct answer is **B**.

**5.** Three fourths of a pitcher is filled with pineapple juice. The pitcher is emptied by pouring an equal amount of juice into each of $5$ cups. What percent of the total capacity of the pitcher did each cup receive?

- A $5$
- B $10$
- C $15$
- D $20$
- E $25$

## Solution(s):

Since we start with $\frac{3}{4}$ of the pitcher, we have $75\%$ of the pitcher full as

$$\frac{3}{4} = \frac{75}{100}.$$

Now, since $5$ cups each have the same amount of juice, they each have one-fifth of the $75\%$.

This means they have

$$\frac{1}{5} \cdot 75\% = \left(\frac{75}{5}\right)\% = 15\%.$$

Thus, the correct answer is **C**.

**6.** Aaron, Darren, Karen, Maren, and Sharon rode on a small train that has five cars that seat one person each. Maren sat in the last car. Aaron sat directly behind Sharon. Darren sat in one of the cars in front of Aaron. At least one person sat between Karen and Darren. Who sat in the middle car?

A Aaron

B Darren

C Karen

D Maren

E Sharon

## Solution(s):

We know that they must be arranged as such: \text{__, __, __, __, Maren}, with each "__" representing people whose location we don't yet know.

Note that Maren must be there since he sat in the last car.

Since we know Sharon sat in front of Aaron, we can take cases of the possible locations of them both.

Case 1: \text{Sharon, Aaron, __, __, Maren} Since Darren sits in front of Aaron, we can't find a space for him so this case can't work.

Case 2: \text{__, Sharon, Aaron, __, Maren} Since Darren sits in front of Aaron, we know the configuration must be Darren, Sharon, Aaron, __, Maren -- leaving one spot for Karen.

The final configuration is Darren, Sharon, Aaron, Karen, Maren.

Case 3: \text{__, __, Sharon, Aaron, Maren} This case would involve Darren and Karen being next to each other, which contradicts the condition that at least one person is between them.

Since the only valid configuration is Darren, Sharon, Aaron, Karen, Maren, we know Aaron is in the middle.

Thus, the correct answer is **A**.

**7.** How many integers between $2020$ and $2400$ have four distinct digits arranged in increasing order? (For example, $2357$ is one integer.)

- A $9$
- B $10$
- C $15$
- D $21$
- E $28$

## Solution(s):

Since the integers are between $2020$ and $2400$, we know the thousands digit must be 2 and the hundreds digit must be between $0$ and $3$.

Since the digits are increasing, the second digit must be greater than $2$, so it can only be $3$.

This means the tens and units digits are different digits each greater than or equal to $4$.

Suppose we choose 2 distinct digits each greater than or equal to $4$. There are $6$ digits for the first choice and $5$ digits for the second choice.

This means there are $30$ combinations.

However, we ignore exactly half of these combinations as each combination has an equal likelihood of being ascending or descending.

This leaves $\frac{30}{2} = 15$ combinations.

Thus, the correct answer is **C**.

**8.** Ricardo has $2020$ coins, some of which are pennies ($1$-cent coins) and the rest of which are nickels ($5$-cent coins). He has at least one penny and at least one nickel. What is the difference in cents between the greatest possible and least possible amounts of money that Ricardo can have?

A $8062$

B $8068$

C $8072$

D $8076$

E $8082$

## Solution(s):

Let $p$ be the number of pennies Ricardo has and let $n$ be the number of nickels he has.

We know that

$$p + n = 2020,$$

$$p \geq 1,$$

and

$$n \geq 1$$

by the problem statement.

This means

$$\begin{aligned} & 2020 - n = p \\ \implies & 2020 - n \geq 1. \end{aligned}$$

Therefore, $1 \leq n \leq 2019$. It follows, then, that Ricardo has

$$\begin{aligned} p + 5n &= p + n + 4n \\ &= 2020 + 4n \end{aligned}$$

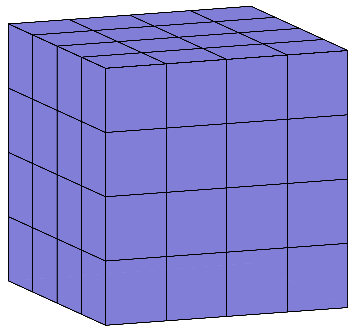
cents.

Therefore, to maximize the money he has, we maximize the number of nickels he has, and minimizing the money he has involves minimizing the number of nickels he has. The maximum number of nickels he can have is $2019$, so he can have at most $2020 + 2019(4)$ cents. The minimum number of nickels he can have is $1$, so he has at most $2020 + 1(4)$ cents. The difference between the maximum and minimum amount of money he can have is:

$$\begin{gathered} 2020 + 2019(4) - (2020 + 1(4)) \\ = 2018(4) = 8072. \end{gathered}$$

Thus, the correct answer is **C**.

**9.** Akash's birthday cake is in the form of a $4 \times 4 \times 4$ inch cube. The cake has icing on the top and the four side faces, and no icing on the bottom. Suppose the cake is cut into $64$ smaller cubes, each measuring $1 \times 1 \times 1$ inch, as shown below. How many small pieces will have icing on exactly two sides?



A $12$

B $16$

C $18$

D $20$

E $24$

## Solution(s):

On the top side, there are $4$ cubes that are only on the top. Also, there are $4$ cubes that have icing on $3$ sides. Therefore, we have $16 - 4 - 4 = 8$ cubes on the top that have icing on exactly $2$ sides.

For the $4$ other sides, we need to find the number of cubes that have icing on exactly two sides, exluding the cubes we counted on the sides. Each face has $6$ cubes on the edge that have icing on exactly two sides. However, this would double count when taking into account the other faces, as each cube would be counted for two faces. Therefore, we need to add $\dfrac{6 \cdot 4}{2} = 12$ cubes.

The final answer is $8 + 12 = 20$.

Thus, the correct answer is **D**.

**10.** Zara has a collection of $4$ marbles: an Aggie, a Bumblebee, a Steelie, and a Tiger. She wants to display them in a row on a shelf, but does not want to put the Steelie and the Tiger next to one another. In how many ways can she do this?

A $6$

B $8$

C $12$

D $18$

E $24$

## Solution(s):

Let $X$ and $Y$ represent the Steelie and the Tiger in some order, with $X$ always coming before $Y$.

To place $X$ and $Y$ we can have the following cases:

$X, \_\_, Y, \_\_$ ;

$X, \_\_, \_\_, Y$;

$\_\_, X, \_\_, Y, .$

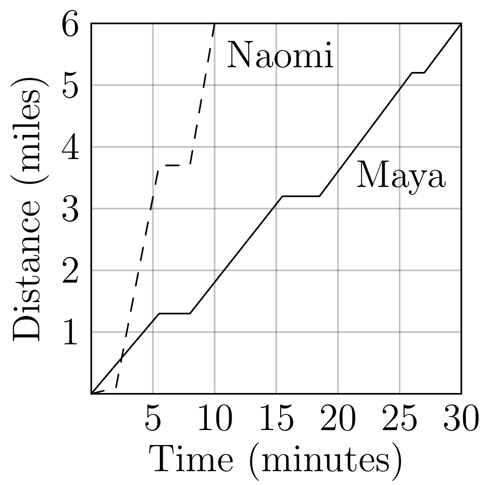
leaving 3 configurations, with "__" being the reserved spots for the other marbles. With any other configuration, we have $X$ and $Y$ next to each other or $Y$ before $X$.

Now, each configuration can have either the Steelie be $X$ and the Tiger be $Y$ or Steelie be $Y$ and the Tiger be $X$. This doubles the number of configurations we have, making $6$.

Also, each configuration can have either the Bumblebee be the first spot and Aggie be the second spot or vice versa. This again doubles the number of configurations we have, making $12$.

Thus, the correct answer is **C**.

**11.** After school, Maya and Naomi headed to the beach, 6 miles away. Maya decided to bike while Naomi took a bus. The graph below shows their journeys, indicating the time and distance traveled. What was the difference, in miles per hour, between Naomi's and Maya's average speeds?




A 6

B 12

C 18

D 20

E 24

## Solution(s):

Naomi traveled 6 miles in 10 minutes. This ratio is

$$\frac{6 \text{ miles}}{10 \text{ minutes}} = \frac{36 \text{ miles}}{60 \text{ minutes}}$$
$$= 36 \frac{\text{miles}}{\text{hour}}$$

Maya traveled 6 miles in 30 minutes. This ratio is

$$\frac{6 \text{ miles}}{30 \text{ minutes}} = \frac{12 \text{ miles}}{60 \text{ minutes}}$$
$$= 12 \frac{\text{miles}}{\text{hour}}$$

This difference is $36 - 12 = 24.$

Thus, the correct answer is **C**.

**12.** For a positive integer $n$, the factorial notation $n!$ represents the product of the integers from $n$ to $1$. For example:

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

What value of $N$ satisfies the following equation?

$$5! \times 9! = 12 \times N!$$

- A $10$
- B $11$
- C $12$
- D $13$
- E $14$

## Solution(s):

Note first that

$$\begin{aligned} n! &= n \cdot (n-1) \cdot (n-2) \cdots 1 \\ &= n((n-1) \cdot (n-2) \cdots 1) \\ &= n(n-1)! \end{aligned}$$

With that in mind, further observe that:

$$\begin{aligned} 5! \cdot 9! &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 9! \\ &= 120 \cdot 9! \\ &= 12(10 \cdot 9!) \end{aligned}$$

Since $12 \cdot N! = 12(10 \cdot 9!)$, we know $N! = 10 \cdot 9!$.

Using our note from above, we know that $10 \cdot 9! = 10!$ so $N = 10$.

Thus, the correct answer is **A**.

**13.** Jamal has a drawer containing $6$ green socks, $18$ purple socks, and $12$ orange socks. After adding more purple socks, Jamal noticed that there is now a $60\%$ chance that a sock randomly selected from the drawer is purple. How many purple socks did Jamal add?

A $6$

B $9$

C $12$

D $18$

E $24$

## Solution(s):

Suppose Jamal adds $s$ purple socks. Then, there will be $s+18$ purple socks.

Also, since there are $36$ total socks to begin with, we have $s+36$ socks after adding the socks.

Since we have a $60\%$ chance of choosing a purple sock afterwards, we know

$$\frac{s+18}{s+36}=0.6.$$

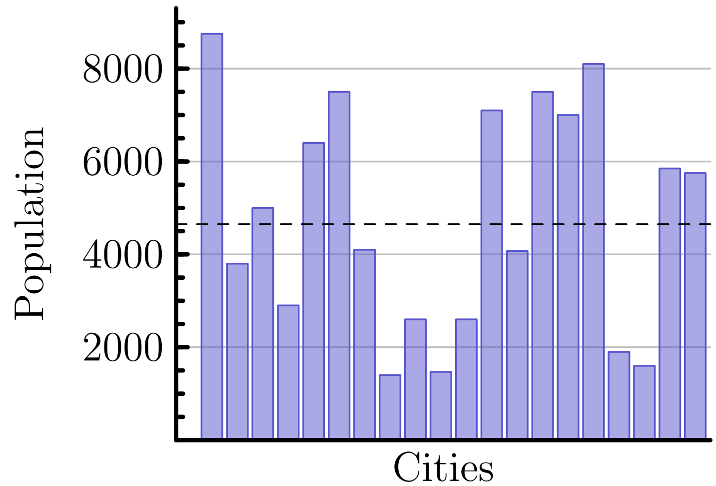
Solving for $s$ yields:

$$\begin{aligned} s+18 &= 0.6s+21.6 \\ 0.4s &= 3.6 \\ s &= 9. \end{aligned}$$

Therefore, $9$ socks are added.

Thus, the correct answer is **B**.

**14.** There are $20$ cities in the County of Newton. Their populations are shown in the bar chart below. The average population of all the cities is indicated by the horizontal dashed line. Which of the following is closest to the total population of all $20$ cities?




A $65,000$

B $75,000$

C $85,000$

D $95,000$

E $105,000$

## Solution(s):

Looking at the horizontal dashed line, the average population is around $4750$.

Since there are $20$ cities, and the average population is $\frac{\text{total population}}{\text{number of cities}}$, we know:

$$\frac{\text{total population}}{20} = 4750.$$

Multiplication yields that the total population is $95,000$.

Thus, the correct answer is **D**.

**15.** Suppose $15\%$ of $x$ equals $20\%$ of $y$. What percentage of $x$ is $y$?

- A $5$
- B $35$
- C $75$
- D $133\frac{1}{3}$
- E $300$

## Solution(s):

If a number $m$ is $p$ percent of $n$ then $\frac{p}{100}m = n$.

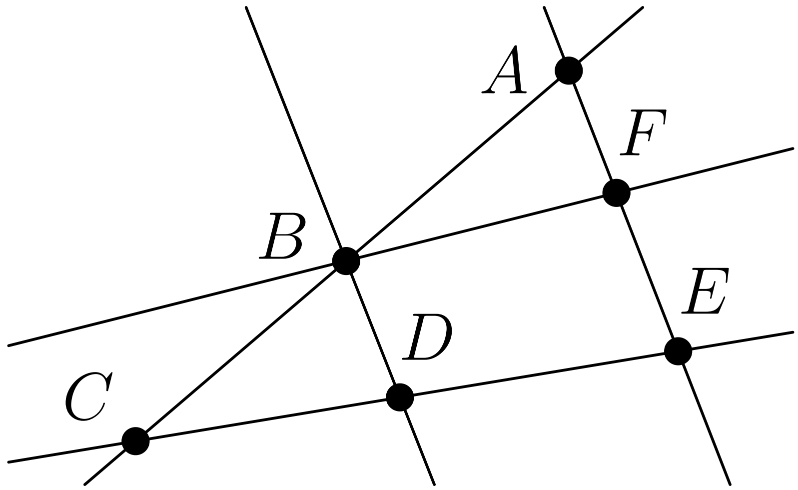
This means that $0.15x = 0.2y$.

Dividing each side by $0.2$ gives that $0.75x = y$.

Therefore, we know that $x$ is $75\%$ of $y$.

Thus, the correct answer is **C**.

**16.** Suppose Each of the points $A, B, C, D, E,$ and $F$ in the figure below represents a different digit from $1$ to $6$. Each of the five lines shown passes through some of these points. The digits along each line are added to produce five sums, one for each line. The total of the five sums is $47$. What is the digit represented by B?




A $1$

B $2$

C $3$

D $4$

E $5$

## Solution(s):

Each number is added once per line it is on. Every point is on $2$ lines except for $B$ which is on $3$.

This means

$$2A + 3B + 2C + 2D + 2E + 2F = 47,$$

so

$$2(A + B + C + D + E + F) = 47 - B.$$

Since $A, B, C, D, E, F$ are unique digits from $1$ to $6$, each digit is represented exactly once, making

$$A + B + C + D + E + F \\ = 21.$$

With

$$2(A + B + C + D + E + F) \\ + B = 47,$$

we know $B + 2(21) = 47$, so $B = 5$.

Thus, the correct answer is **E**.

**17.** How many factors of $2020$ have more than $3$ factors? (As an example, $12$ has $6$ factors, namely $1, 2, 3, 4, 6,$ and $12$.)

A $6$

B $7$

C $8$

D $9$

E $10$

## Solution(s):

Let's begin by firstly simply factoring $2020$:

$$\begin{aligned} 2020 &= 1 \cdot 2020 \\ &= 2 \cdot 1010 \\ &= 4 \cdot 505 \\ &= 5 \cdot 404 \\ &= 10 \cdot 202 \\ &= 20 \cdot 101 \end{aligned}$$

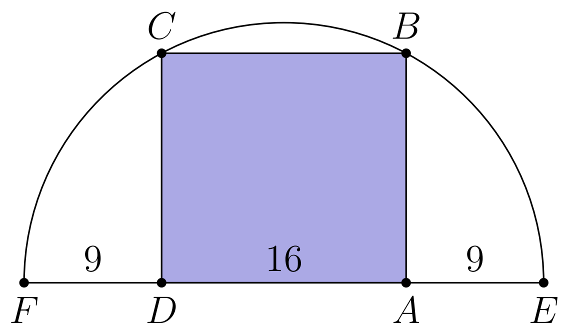
These twelve factors of $2020$ can be classified by the number of their factors:

- $1$ has one factor
- $2, 5$,and $101$ has two factors
- $4$ has three (distinct) factors

Thus, all the remaining seven numbers must have more than three factors.

Thus, the correct answer is **B**.

**18.** Rectangle $ABCD$ is inscribed in a semicircle with diameter $\overline{FE}$, as shown in the figure. Let $DA = 16$, and let $FD = AE = 9$. What is the area of $ABCD$?




A $240$

B $248$

C $256$

D $264$

E $272$

## Solution(s):

Since $FE$ is the diameter of the semicircle, we know the length of the diameter is $34$, and so the radius is $17$. Let $O$ be the center of the diameter.

The length from $OF$ therefore is $17$.

Since $D$ is on $OF$, we know

$$\begin{aligned} OD + FD &= OF \\ OD + 9 &= 17 \\ OD &= 8. \end{aligned}$$

Also, since we have a semicircle, we know $OC = 17$.

Finally, since $ABCD$ is a rectangle, we know $\angle ODC$ is a right angle. This means we can find $DC$ by the Pythagorean Theorem. We know

$$\begin{aligned} OD^2 + DC^2 &= OC^2 \\ 8^2 + DC^2 &= 17^2 \\ DC &= 15. \end{aligned}$$

As such, the area of the rectangle is $DC \cdot DA = 15 \cdot 16 = 240$.

Thus, the correct answer is **A**.

**19.** A number is called flippy if its digits alternate between two distinct digits. For example, $2020$ and $37373$ are flippy, but $3883$ and $123123$ are not. How many five-digit flippy numbers are divisible by $15$?

A $3$

B $4$

C $5$

D $6$

E $8$

## Solution(s):

For a number to be divisible by $15$ the number must be divisible by $3$ and by $5$.

First, to ensure the number is divisible by $5$ it must end in $0$ or in $5$.

Since the digits alternate between two distinct digits, we call the other digit $d$.

This would make our number either $0d0d0$ or $5d5d5$.

We can ignore the numbers in the form $0d0d0$ as they would reduce to a $4$ digit number.

As such, we know our number is $5d5d5$ for some digit $d$.

To ensure our number is also a multiple of $3$ the sum of the digits must be a multiple of $3$.

The sum of our digits is $5 + d + 5 + d + 5 = 15 + 2d$. Since $15$ is a multiple of $3$, all that is required is that $d$ is a multiple of $3$.

This means $d$ can be $0, 3, 6$ or $9$.

Therefore, we have $4$ solutions.

Thus, the correct answer is **B**.

**20.** A scientist walking through a forest recorded as integers the heights of $5$ trees standing in a row. She observed that each tree was either twice as tall or half as tall as the one to its right. Unfortunately some of her data was lost when rain fell on her notebook. Her notes are shown below, with blanks indicating the missing numbers. Based on her observations, the scientist was able to reconstruct the lost data. What was the average height of the trees, in meters?

\renewcommand{\arraystretch}{1.5} \begin{array}{|c|c|} \hline \text{Tree 1} & \text{__ meters} \\ \text{Tree 2} & 11 \text{ meters} \\ \text{Tree 3} & \text{__ meters} \\ \text{ Tree 4 }& \text{__ meters} \\ \text{Tree 5} & \text{__ meters} \\ \hline \text{Average height} & \text{__.2 meters} \\ \hline \end{array}

A $22.2$

B $24.2$

C $33.2$

D $35.2$

E $37.2$

## Solution(s):

We know that tree 2 is $11$ meters tall, and since the trees on either side of any given true were said to be either double the height or half the height, we can conclude that tree $1$ and three $3$ must both be $22$ meters tall, as if they were half that of tree $2$, they would not be integers.

This gives us only four possibilities for the remaining two trees, of which we simply test each case for an average that ends in $".2"$:

- Tree $4$ is $44$ meters; Tree $5$ is $88$ meters:

$$\frac{55+44+88}{5}=37.4$$

- Tree $4$ is $44$ meters; Tree $5$ is $22$ meters:

$$\frac{55+44+22}{5}=24.2$$

- Tree $4$ is $11$ meters; Tree $5$ is $22$ meters:

$$\frac{55+11+22}{5}=17.6$$

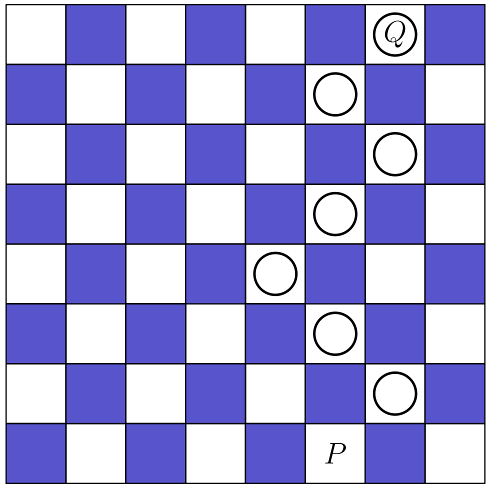
- Tree $4$ is $11$ meters;   Tree $5$ is $5.5$ meters:

  This means that Tree $5$ is not an integer, as therefore this case is invalid and discarded.

The only one of these cases that has an average that ends in $".2"$ is the $11, 22$ case. As such, the average height is $24.2$ meters.

Thus, the correct answer is **B**.

**21.** A game board consists of $64$ squares that alternate between blank and colored. The figure below shows square $P$ in the bottom row and square $Q$ in the top row. A marker is placed at $P$. A step consists of moving the marker onto one of the adjoining blank squares in the row above. How many $7$-step paths are there from $P$ to $Q$? (The figure shows a sample path.)




A $28$

B $30$

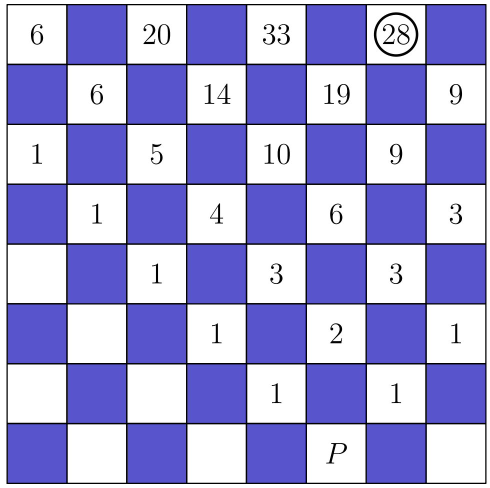
C $32$

D $33$

E $35$

## Solution(s):

Every move must go either down or up and either left or right. Since the final position is $7$ moves up and we move $7$ times, every move must go up. Since after $n$ moves, we are on the $n$th row, we can use the $n$th row to construct the ways to get to each position from the previous moves. Each move can come from the down-left or down-right direction, so we get the ways to get to a point by adding the number of paths from those two directions.




We have then constructed the ways to get to each point from $P$ assuming we always go up. The circled point is $Q$, so we have $28$ ways to get to $Q$.

Thus, the correct answer is **A**.

**22.** When a positive integer $N$ is fed into a machine, the output is a number calculated according to the rule shown below.

$$N \mapsto \begin{cases} \frac{N}{2}, N \text{ is even} \\ 3N+1, N \text{ is odd} \end{cases}$$

For example, starting with an input of $N = 7$, the machine will output $3 \cdot 7 + 1 = 22$. Then if the output is repeatedly inserted into the machine five more times, the final output is $26$.

$$\begin{gather*} 7 \to 22 \to 11 \to 34 \\ \to 17 \to 52 \to 26 \end{gather*}$$

When the same $6$-step process is applied to a different starting value of $N$, the final output is $1$. What is the sum of all such integers $N$? \begin{gather*} N \to \text{ __ } \to \text{ __ } \to \text{ __ } \\ \to \text{ __ } \to \text{ __ } \to 1 \end{gather*}

- A $73$
- B $74$
- C $75$
- D $82$
- E $83$

## Solution(s):

To see which numbers we can make by inverting it, let's make an inverting machine.

This would take $N$ and yield either $2N$ if $2N$ is even (which it always is) or $\frac{N-1}{3}$ if $\frac{N-1}{3}$ is an odd integer. Note that $\frac{N-1}{3}$ is an integer only if $N \equiv 1 \mod 3$. Also, if $N$ is even, then $N-1$ is odd. That would mean $\frac{N-1}{3}$ would be odd. Therefore, our inverter machine yields $2N$ and also $\frac{N-1}{3}$ if $N \equiv 1 \mod 3$ and $N$ is even.

Now, we must see what the inverting machine can yield after $6$ moves:

$1)$ We can only get $2$.

$2)$ From $2$, we can only get $4$.

$3)$ From $4$, we can get $1$ and $8$.

$4)$ From $1$, we can get only $2$; from $8$, we can only get $16$.

$5)$ From $2$, we can get only $4$; from $16$, we can get $5$ and $32$.

$6)$ From $4$, we can get $1$ and $8$,; $5$, we can get $10$; from $32$ we can get $64$.

Move $6$ can yield $1, 8, 10$, and $64$ and their sum is $83$.

Thus, the correct answer is **E**.

**23.** Five different awards are to be given to three students. Each student will receive at least one award. In how many different ways can the awards be distributed?

A $120$

B $150$

C $180$

D $210$

E $240$

## Solution(s):

First, we can calculate the number of way to just give out $5$ awards, without making sure every student has at least $1$. This is $3^5 = 243$.

Next we subtract the number of distributions without everyone having at least $1$.

This can be counted as the number of distributions with at most $2$ people having every award.

We can sort this by two cases. Case 1 has exactly 2 people having at least one award. Case 2 has exactly 1 person having at least one award.

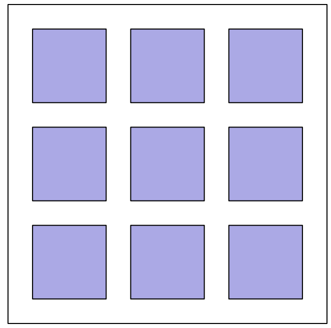
Case 1: If two people have all the awards, there are $2^5$ ways to distribute the awards amongst these two people. However, we must subtract $2$ as that has only one person owning all the awards, which is outside the definiton of this case. This has $30$ combinations. Finally, we multiply this by $3$ as there are $3$ ways to choose the group of two people. This case yields $90$ distributions

Case 2: If one people has all the awards, there is $1$ way to distribute the awards amongst that person. Finally, we multiply this by $3$ as there are $3$ ways to choose the person. This case yields $3$ distributions.

There are a total of $93$ cases to remove. Therefore, the answer is $243 - 93 = 150$.

Thus, the correct answer is **B**.

**24.** A large square region is paved with $n^2$ gray square tiles, each measuring $s$ inches on a side. A border $d$ inches wide surrounds each tile. The figure below shows the case for $n=3$. When $n=24$ , the $576$ gray tiles cover $64\%$ of the area of the large square region. What is the ratio $\frac{d}{s}$ for this larger value of $n$?



A $\dfrac{6}{25}$

B $\dfrac{1}{4}$

C $\dfrac{9}{25}$

D $\dfrac{7}{16}$

E $\dfrac{9}{16}$

## Solution(s):

First, since $n=24$, there are $n^2=24^2=576$ squares.

Since we aren't given an area of the large square and only the ratios within it, we can define the area to be $1$. Since the total area of the gray is $\dfrac{64}{100}=(\dfrac{4}{5})^2$, the area for each square is

$$\left(\frac{4}{5}\right)^2\left(\frac{1}{576}\right)$$
$$=\left(\frac{4}{5}\right)^2\left(\frac{1}{24}\right)^2=\left(\frac{1}{30}\right)^2.$$

Therefore, the side length of each tile is:

$$\sqrt{\left(\frac{1}{30}\right)^2} = \frac{1}{30}.$$

Now, there are $n = 24$ tiles and $n + 1 = 25$ borders, so, $25d + 24s = 1$.

Since $s = 24$, we have

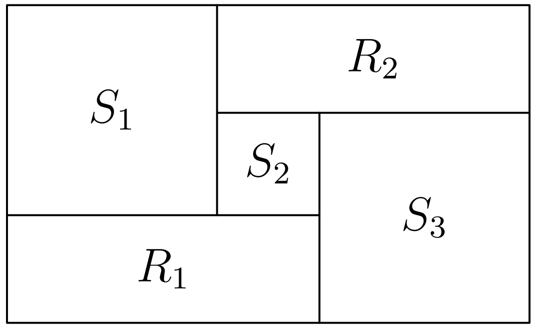
$$\begin{aligned} 25d + 24\left(\frac{1}{30}\right) &= 1 \\ 25d &= \frac{1}{5} \\ d &= \frac{1}{125}. \end{aligned}$$

Finally,

$$\begin{aligned} \frac{d}{s} &= \frac{\frac{1}{125}}{\frac{d}{30}} \\ &= \frac{30}{125} \\ &= \frac{6}{25}. \end{aligned}$$

Thus, the correct answer is **A**.

**25.** Rectangles $R_1$ and $R_2$, and squares $S_1$, $S_2$, and $S_3$, shown below, combine to form a rectangle that is 3322 units wide and 2020 units high. What is the side length of $S_2$ in units?




A $651$

B $655$

C $656$

D $662$

E $666$

## Solution(s):

We represent the lengths of each square as $s_1, s_2$, and $s_3$ respectively. The length of the rectangle is $s_1 + s_2 + s_3$ as these $3$ squares span the entirety of a side of the large rectangle. Therefore,

$$s_1 + s_2 + s_3 = 3322.$$

Also, the height of the large rectangle is the sum of the height of $R_2$ and $S_3$. Now, note that the sum of height of $R_2$ and $s_2$ is $s_1$, so height of $R_2$ is equal to $s_1 - s_2$. Therefore, the height of the large rectangle is $s_1 - s_2 + s_3$, which means

$$s_1 + s_2 + s_3 = 2020.$$

Subtracting both of our results yields

$$2s_2 = 3322 - 2020 = 1302.$$

This would mean

$$s_2 = 651.$$

Thus, the correct answer is **A**.

Problems: https://live.poshenloh.com/past-contests/amc8/2020