2022 AMC 8 Solutions

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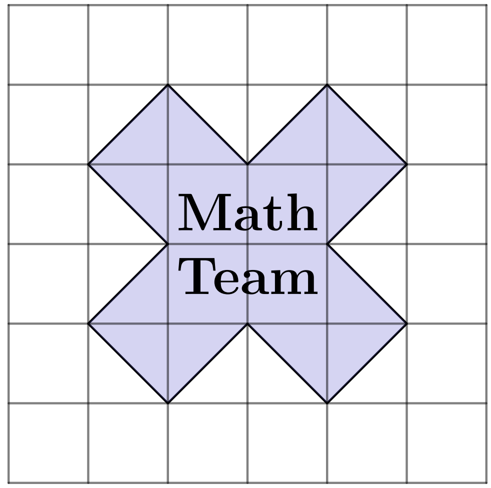
https://live.poshenloh.com/past-contests/amc8/2022/solutions

1. The Math Team designed a logo shaped like a multiplication symbol, shown below on a grid of 1-inch squares. What is the area of the logo in square inches?



A 10

B 12

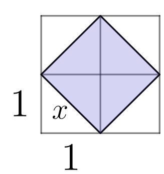
C 13

D 14

E 15

Solution(s):

Let's first consider the following:



Using the Pythagorean theorem, we know how to solve for x relatively easily:

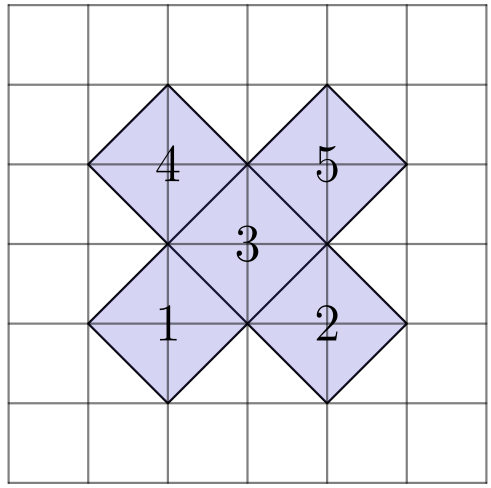
$$a^2 + b^2 = c^2 \iff 1^2 + 1^2 = x^2$$

$$\iff x = \sqrt{2}$$

With that in mind, let's try and find the area of the purple square:

$$A = x^2 = (\sqrt{2})^2 = 2$$

We now know that the area of this slanted, small square is equal to 2. With that in mind, let's look at the full picture:



We can clearly see that there are 5 of these same small purple squares that make up the logo. We already know that the area of one of the small purple squares is equal to 2, so the total area A_t is:

$$A_t = 5(A) = 5(2) = 10$$

Thus, the answer is **A**.

2. Consider these two operations:

$$\begin{aligned} a \blacklozenge b &= a^2 - b^2 \\ a \star b &= (a-b)^2 \end{aligned}$$

Compute the value:

$$(5 \blacklozenge 3) \star 6$$

A -20

B 4

C 16

D 100

E 220

Solution(s):

Given the definitions of $\blacklozenge$ and $\star$, we can directly substitute these operations for said definitions to get:

$$\begin{aligned} (5 \blacklozenge 3) \star 6 &= (5^2 - 3^2) \star 6 \\ &= 16 \star 6 \\ &= (16-6)^2 \\ &= 100 \end{aligned}$$

Thus, the answer is **D**.

3. When three positive integers a, b, and c are multiplied together, their product is 100. Suppose $a < b < c$. In how many ways can the numbers be chosen?

A 0

B 1

C 2

D 3

E 4

Solution(s):

We are given that $a < b < c, abc = 100$.

As such, we know that $a^3 < abc = 100$, which implies that $a^3 < 100$.

Considering $\sqrt[3]{100}$, we know that $4^3 = 64 < 100$ and $5^3 = 125 > 100$, and therefore, we can conclude that $a \geq 4$. As a is a positive integer, we have four possible values of $a : 1, 2, 3, 4$. As a is a factor of 100, we can eliminate the possibility of a being 3, and are now left with three cases: $a = 1, 2, 4$.

If $a = 1$, then $bc = 100$. Since $b < c$, we know $b^2 < bc = 100$ so $1 < b < 10$. Since b is a factor of 100 that satisfies this constraint, b must be either $2, 4, 5$. This creates the solutions of $(1, 2, 50), (1, 4, 25)$, or $(1, 5, 20.)$

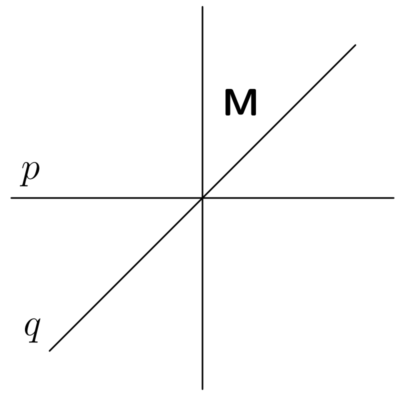
If $a = 2$, then $bc = 50$. Since $b < c$, we know $b^2 < bc = 50$ so $2 < b < 8$. Since b is a factor of 50 that satisfies this constraint, b must be 5. This creates the solution of $(2, 5, 10.)$

If $a = 4$, then $bc = 25$. Since $b < c$, we know $b^2 < bc = 25$ so $4 < b < 5$. This creates no integer solutions for b.

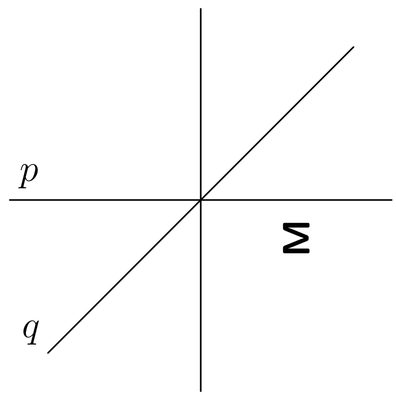
We therefore have 4 total solutions.

Thus, the answer is **A**.

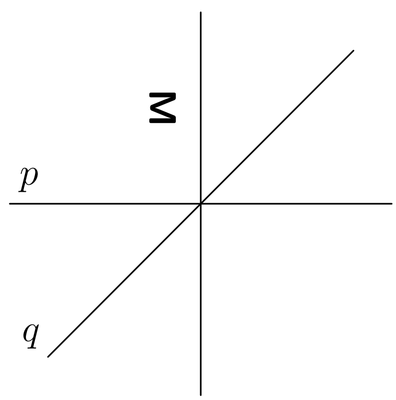
4. The letter **M** in the figure below is first reflected over the line q and then reflected over the line p. What is the resulting image?



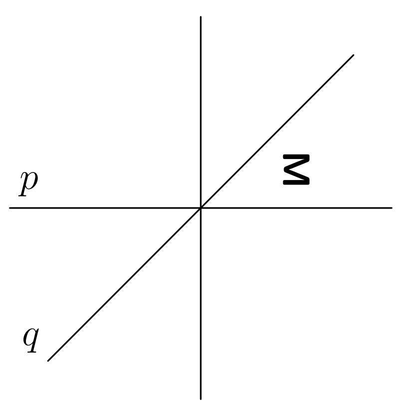
A



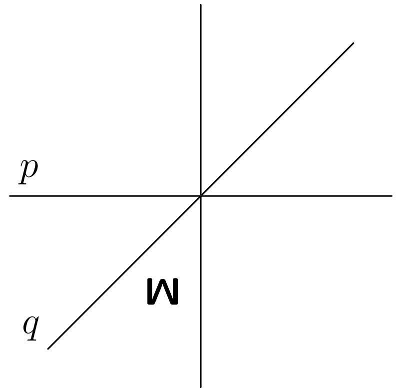
B



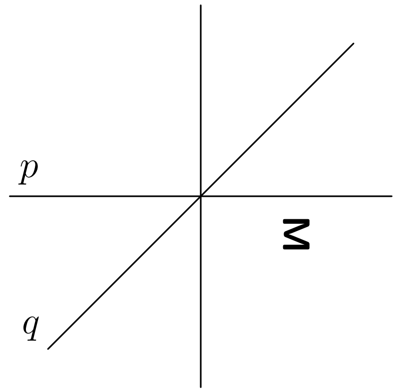
C



D



E

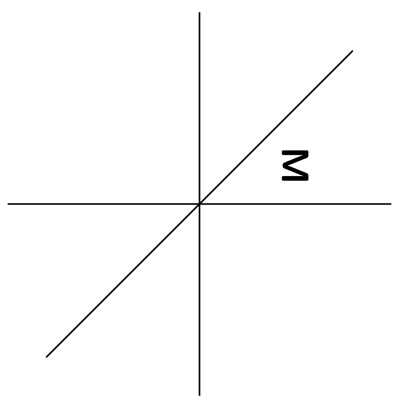


Solution(s):

First reflect

M

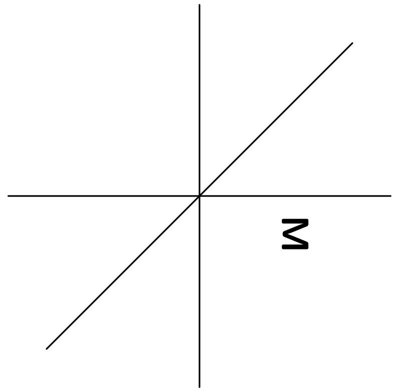
over the line q, from which we obtain the following:



Then reflect

M

over the line p, from which we obtain the following:



Thus, the answer is **E**.

5. Anna and Bella are celebrating their birthdays together. Five years ago, when Bella turned 6 years old, she received a newborn kitten as a birthday present. Today the sum of the ages of the two children and the kitten is 30 years. How many years older than Bella is Anna?

A 1

B 2

C 3

D 4

E 5

Solution(s):

If Bella was 6 five years ago, then she is 11 right now.

If the kitten was a newborn five years ago, then it is 5 right now.

Since the sum of all 3 ages is 30, Anna's age is

$$30-5-11=14$$

Since Anna is 14 and Bella is 11, she is 3 years older than Bella.

Thus, the answer is **C**.

6. Three positive integers are equally spaced on a number line. The middle number is 15 and the largest number is 4 times the smallest number. What is the smallest of these three numbers?

A 4

B 5

C 6

D 7

E 8

Solution(s):

Since one of the outer number is 4 times the other, we can make one of the numbers x and the other be $4x$.

Since the numbers are equally spaced, the middle number is the average of the outer two.

This means the average of x and $4x$ is 15, so $\frac{x+4x}{2} = 15$. This means $5x = 30$, so $x = 6$. Our outer numbers therefore are $6, 24$, so the smaller one is 6.

Thus, the answer is **C**.

7. When the World Wide Web first became popular in the 1990s, download speeds reached a maximum of about 56 kilobits per second. Approximately how many minutes would the download of a 4.2-megabyte song have taken at that speed? (Note that there are 8000 kilobits in a megabyte.)

- A 0.6
- B 10
- C 1800
- D 7200
- E 36000

Solution(s):

Given our definition of a megabyte as 8000 kilobits, we know 4.2 megabytes is $8000 \cdot 4.2$ kilobits. This can be rewritten as $(8000 \cdot \frac{3}{40}) \cdot (4.2 \cdot \frac{40}{3})$ kilobits which leads to us having $600 \cdot 56$ kilobits. Since we know there are 56 kilobits per second, we have 600 seconds. This leads us to the answer of 10 minutes.

Thus, the answer is **B**.

8. What is the value of:

$$\frac{1}{3}\cdot\frac{2}{4}\cdot\frac{3}{5}\cdots\frac{18}{20}\cdot\frac{19}{21}\cdot\frac{20}{22}$$

A $\frac{1}{462}$

B $\frac{1}{231}$

C $\frac{1}{132}$

D $\frac{2}{213}$

E $\frac{1}{22}$

Solution(s):

Since every integer from 3 to 20 occurs once as a denominator and once as a numerator, they cancel each other out.

After canceling every number out, we have only 1 and 2 left as numerators and 21 and 22 left as denominators.

The remaining fraction is $\frac{1\cdot 2}{21\cdot 22}$. This simplifies to $\frac{2}{462}=\frac{1}{231}$

9. A cup of boiling water (212° F) is placed to cool in a room whose temperature remains constant at 68° F. Suppose the difference between the water temperature and the room temperature is halved every 5 minutes. What is the water temperature, in degrees Fahrenheit, after 15 minutes?

A 77

B 86

C 92

D 98

E 104

Solution(s):

The current difference is $212 - 68 = 144$.

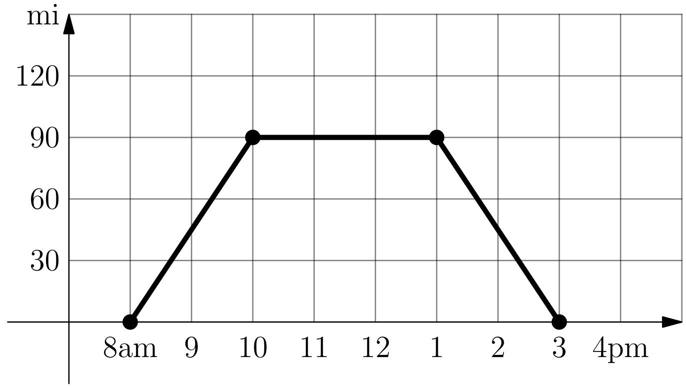
Since we have 15 minutes while halving every 5 minutes, we halve the difference 3 times.

This means the difference is multiplied by $(\frac{1}{2})^3 = \frac{1}{8}$, so our new difference is $\frac{1}{8} \cdot 144 = 18$. This makes our final temperature $68 + 18 = 86$.

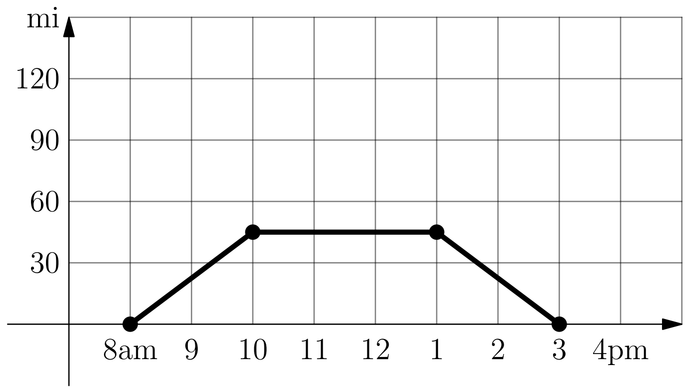
Thus, the answer is **B**.

10. One sunny day, Ling decided to take a hike in the mountains. She left her house at 8 a.m., drove at a constant speed of 45 miles per hour, and arrived at the hiking trail at 10 a.m. After hiking for 3 hours, Ling drove home at a constant speed of 60 miles per hour. Which of the following graphs best illustrates the distance between Ling's car and her house over the course of her trip?

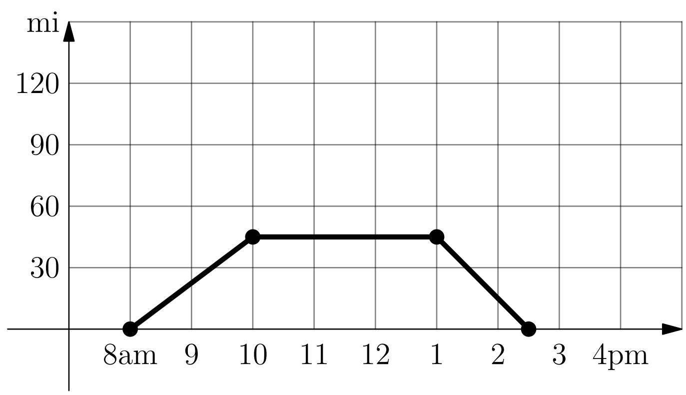
A



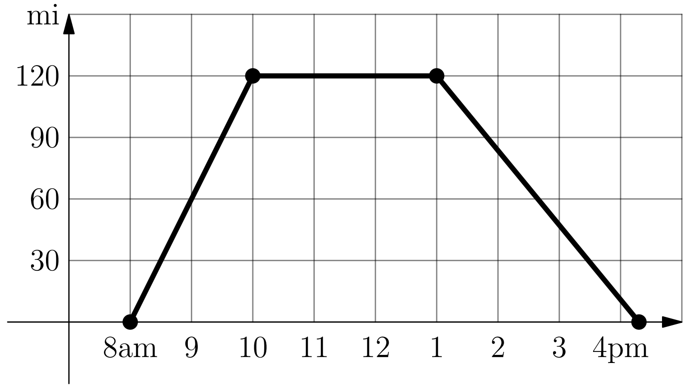
B



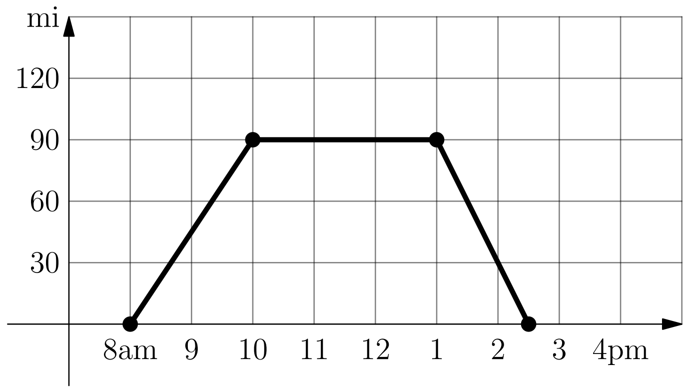
C



D



E



Solution(s):

She drives 45 miles per hour for the first 2 hours, so she travels 90 miles in the first two hours. This leaves choices A, E.

Since she hikes for 3 hours, she starts going back at 1pm.

She drives 90 miles at 60 mph, so she gets back in $\frac{90}{60} = 1.5$ hours, so she is back at $2{:}30$.

Thus, the answer is **E**.

11. Henry the donkey has a very long piece of pasta. He takes a number of bites of pasta, each time eating 3 inches of pasta from the middle of one piece. In the end, he has 10 pieces of pasta whose total length is 17 inches. How long, in inches, was the piece of pasta he started with?

A 34

B 38

C 41

D 44

E 47

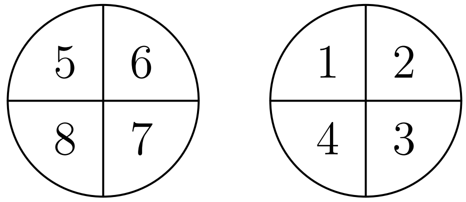
Solution(s):

Since there are 10 pieces, there were 9 locations where a bite was made. Since we have 9 bites and 3 inches are removed per bite, a total of 27 inches were removed.

With 27 inches removed and 17 inches remaining, we know we started with $27 + 17 = 44$ inches.

Thus, the answer is **D**.

12. The arrows on the two spinners shown below are spun. Let the number N equal 10 times the number on Spinner A, added to the number on Spinner B. What is the probability that N is a perfect square number?



Spinner A   Spinner B

A $\frac{1}{16}$

B $\frac{1}{8}$

C $\frac{1}{4}$

D $\frac{3}{8}$

E $\frac{1}{2}$

Solution(s):

This is the same problem as if we made the spinner A make the tens digit and spinner B make the ones digit.

If the tens digit is 5,6,7, or 8, and the ones digit is 1,2,3, or 4, the only possible ways to get a perfect square is 6,4 or 8,1.

There are 4x4=16 possible combinations that are equally likely, and 2 of them satisfy the property. Therfore, our answer is $\frac{2}{16} = \frac{1}{8}$

Thus, the answer is **B**.

13. How many positive integers can fill the blank in the sentence below?

"One positive integer is ____ more than twice another, and the sum of the two numbers is 28"

A 6

B 7

C 8

D 9

E 10

Solution(s):

Let the first number be x. Then the second number is $2x + c$ where both x and c are positive integers.

Since their sum is 28, we know $3x + c = 28$.
This also means $c = 28 - 3x$

With $x > 0$, the smallest possible value of x is 1.

With $c > 0$, we know $28 - 3x > 0$ so $\frac{28}{3} > x$.

This means the maximum value of x is 9.

Since x can be any number from 1 to 9, we have 9 solutions.

Thus, the answer is **D**.

14. In how many ways can the letters in **BEEKEEPER** be rearranged so that two or more **E**'s do not appear together?

A 1

B 4

C 12

D 24

E 120

Solution(s):

First, I claim that every **E** must be in an odd position.
This is because bringing any two **E**s together would bring them right next to each other.

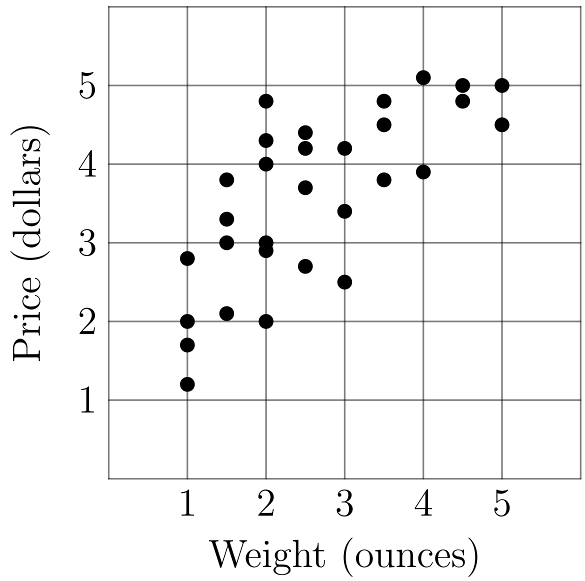
This means the **B,K,P,** and **R** must each be in one of the even positions.

There are 4 choices for a letter in the second position, 3 choices for position for **B**, 3 remaining choices for a position in **K**, 2 remaining choices for a position in **P**, and 1 remaining choice for a position in **R**.

Therefore, there are $4 \cdot 3 \cdot 2 \cdot 1 = 24$ possible choices.

Thus, the answer is **D**.

15. László went online to shop for black pepper and found thirty different black pepper options varying in weight and price, shown in the scatter plot below. In ounces, what is the weight of the pepper that offers the lowest price per ounce?



A 1

B 2

C 3

D 4

E 5

Solution(s):

For each weight, we find the lowest price.

At 1 ounce, we have a price slightly over 1 dollar, so the price per ounce is greater than 1.

At 2 ounces, we have a price of 2 dollars, so the price per ounce is 1.

At 3 ounces, we have a price of about 2.5 dollars, so the price per ounce close to $\frac{2.5}{3}$ which is close to 0.83.

At 4 ounces, we have a price close to 3.9 dollars, so the price per ounce close to $\frac{3.9}{4}$ which is close to 0.97.

At 5 ounces, we have a price close to 4.5 dollars, so the price per ounce close to $\frac{4.5}{5}$ which is close to 0.90.

The price per ounce at 3 ounces is the lowest.

Thus, the answer is **C**.

16. Four numbers are written in a row. The average of the first two is 21, the average of the middle two is 26, and the average of the last two is 30. What is the average of the first and last of the numbers?

A 24

B 25

C 26

D 27

E 28

Solution(s):

Let the numbers be a, b, c, d in order.

Since the average of a and b is 21, we know their sum is $21 \cdot 2 = 42$.
Since the average of c and d is 30, we know their sum is $30 \cdot 2 = 60$.
Since $a + b = 42$ and $c + d = 60$, we know

$$\begin{aligned} a + b + c + d &= 42 + 60 \\ &= 112 \end{aligned}$$

Now, with the average of b and c being 26, we know their sum is $26 \cdot 2 = 52$. This means $b + c = 52$.

Subtracting this result from the sum of all the terms yields $a + d = 50$.
Since $\frac{a+d}{2} = 25$, our answer is 25.

Thus, the answer is **B**.

17. If n is an even positive integer, the double factorial notation $n!!$ represents the product of all the even integers from 2 to n. For example:

$$8!! = 2 \times 4 \times 6 \times 8$$

What is the units digit of the following sum?

$$2!! + 4!! + \cdots + 2022!!$$

A 0

B 2

C 4

D 6

E 8

Solution(s):

If we take $n!!$ for an even n that is greater or equal to 10, then 10 is one of the numbers we multiply by. Since 10 is a factor of $n!!$, we know that the units digit of 10 is 0, which means that it doesn't affect our result.

This means it suffices to compute the units digit of $2!! + 4!! + 6!! + 8!!$, which is equivalent to:

$$\begin{aligned} &2 + 2(4) + 2(4)(6) \\ &+ 2(4)(6)(8) \\ &= 2 + 8 + 48 + 384 \\ &= 442 \end{aligned}$$

The units digit therefore is 2

Thus, the answer is **B**.

18. The midpoints of the four sides of a rectangle are:

$$(-3, 0),$$

$$(2, 0),$$

$$(5, 4),$$

$$(0, 4).$$

What is the area of the rectangle?

A 20

B 25

C 40

D 50

E 80

Solution(s):

Allow:

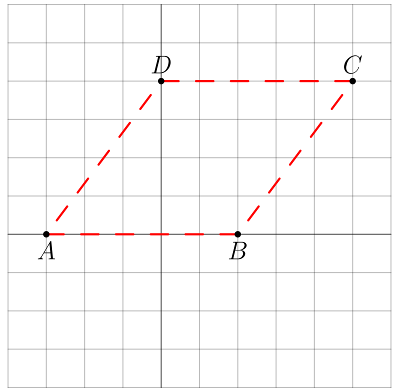
$$A = (-3, 0)$$

$$B = (2, 0)$$

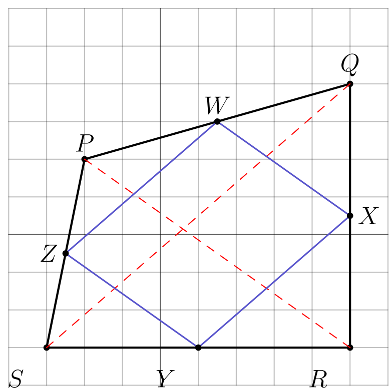
$$C = (5, 4)$$

$$D = (0, 4)$$

This gives us the following rhombus:



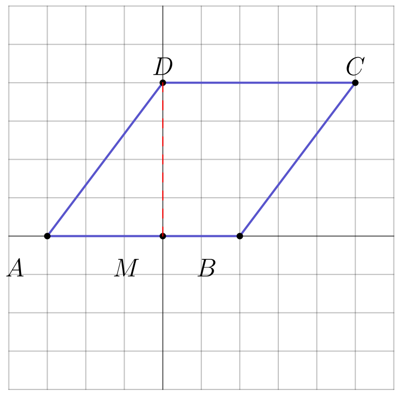
Now, moving away from the rhombus in question, let's consider more generally the relationship between a quadrilateral and the figure formed by its midpoints. Observe the arbitrary quadrilateral $PQRS$, its diagonals (dashed red), and the figure formed by its midpoints:



Note that the actual coordinates of P, Q, R, S don't matter, rather, as long as $PQRS$ is a convex quadrilateral.

Let's first consider the triangle $\triangle QSP$. Notice that $\overline{PW} = \frac{1}{2}\overline{PQ}$ as W is the midpoint of $\overline{PQ}$, and similarly, $\overline{PZ} = \frac{1}{2}\overline{PS}$. Also notice that the angle $\angle SPQ$ is contained within both $\triangle QSP$ and $\triangle WZP$. Therefore, we know that the two triangles are similar, and consequently, the side $\overline{WZ}$ and the diagonal $\overline{SQ}$ are parallel, and $\overline{WZ} = \frac{1}{2}\overline{SQ}$. We can use this same reasoning to show that the side $\overline{XY}$ and the diagonal $\overline{SQ}$ are parallel, and $\overline{XY} = \frac{1}{2}\overline{SQ}$. Furthermore, if we consider the triangles formed by the other diagonal PR, we can reapply this reasoning to show that the sides $\overline{WX}, \overline{YZ}$ and the diagonal $\overline{PR}$ are parallel, and $\overline{WX} = \overline{YZ} = \frac{1}{2}\overline{PR}$.

Therefore, $WXYZ$ is a parallelogram where each of the unequal sides' lengths are equal to half the length of the corresponding parallel diagonal. With that in mind, notice that if we were to create a triangle out of W, X, and the midpoint of $\overline{PR}$, then the area of the right part of the parallelogram would be half the area of the new triangle formed, which would be $\frac{1}{4}$ the area of $\triangle PQR$. Similarly, it is clear that the area of the bottom left part of the parallelogram is half the area of $\triangle PRS$. Adding those two parts up shows that the area of $WXYZ$ is half that of $PQRS$.



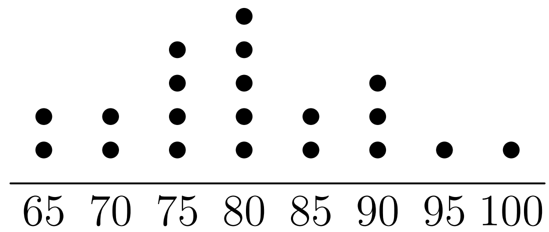
With that in mind, let's go back to the original problem. Calculating the area of $ABCD$:

$$\begin{align} \text{Area} &= \overline{AB} \cdot \overline{MD} \tag{1}\\ &= 5 \cdot 4 \tag{2}\\ &= 20 \tag{3}\end{align}$$

And as the area of $ABCD$ is half that of the quadrilateral whose midpoints created it, we can conclude that the parent quadrilateral has an area of $2 \cdot 20 = 40$.

Thus, the answer is **C**.

19. Mr. Ramos gave a test to his class of 20 students. The dot plot below shows the distribution of test scores.



Later Mr. Ramos discovered that there was a scoring error on one of the questions. He regraded the tests, awarding some of the students 5 extra points, which increased the median test score to 85. What is the minimum number of students who received extra points?

(Note that the median test score equals the average of the 2 scores in the middle if the 20 test scores are arranged in increasing order.)

A 2

B 3

C 4

D 5

E 6

Solution(s):

To make the median equal to 85 the average between the 10th and 11th highest scores must be 85. Therefore, we either have the 10th best and 11th best scores being 85 or having the 10th best score be above 85 and the bottom and the 11th best score be under 85. The second option involves not having any scores of 85.

If we move all the scores out of 85 we have only 7 scores greater than 85. We can't move any other score to be greater than 85. Therefore, this scenario can't happen.

Therefore, we must have the 10th and 11th best scores be 85. There are currently 7 scores greater than or equal to 85, so we must add 4 more scores so the 11th best score is 85.

Thus, the answer is **C**.

20. The grid below is to be filled with integers in such a way that the sum of the numbers in each row and the sum of the numbers in each column are the same. Four numbers are missing. The number x in the lower left corner is larger than the other three missing numbers. What is the smallest possible value of x?

-2	9	5
		-1
x		8

A -1

B 5

C 6

D 8

E 9

Solution(s):

First, by adding the numbers on the top row, we know the sum of the rows and columns are 12.

Since the sum of the numbers in the first column is 12 and we know that one of the numbers is -2, we know the sum of the other two is 14. This means that the number above x is $14 - x$.

Since the sum of the numbers in the bottom row is 12 and we know that one of the numbers is 8, we know the sum of the other two is 4. This means that the number to the right of x is $4 - x$.

Since the sum of the numbers in the middle row is 12 and we know that two of them are $14 - x$ and -1, we know the remaining number is $x - 1$.

We know that $x > 14 - x$ and $x > 4 - x$ as x is the greatest number. This leads us to know $x > 7$. Since $x > x - 1$ is always true, our only restriction is $x > 7$. This means $x = 8$ is our minimum solution.

Thus, the answer is **D**.

21. Steph scored 15 baskets out of 20 attempts in the first half of a game, and 10 baskets out of 10 attempts in the second half. Candace took 12 attempts in the first half and 18 attempts in the second. In each half, Steph scored a higher percentage of baskets than Candace. Surprisingly they ended with the same overall percentage of baskets scored. How many more baskets did Candace score in the second half than in the first?

A 7

B 8

C 9

D 10

E 11

Solution(s):

They both shot 30 baskets total, so if they have the same make percentage, then they made the same total amount. This means Candace also made 25 total shots.

Now, let f be the number of shots Candace made in the first half and let s be the number of shots Candace made in the second half.
We know she made 25 shots, so $f + s = 25$.

By the condition that she had a lower percentage in each half, we know $\frac{f}{12} < \frac{15}{20}$ and $\frac{s}{18} < \frac{10}{10}$. With cross multiplication, we know $f < 9$ and $s < 18$. Since f, s are whole numbers, we have the restriction that $f \leq 8, s \leq 17$.

With our knowledge that $f + s = 25$ and the condition that $f \leq 8, s \leq 17$, the only possible solution is $f = 8$ and $s = 17$.
This makes our answer $s - f = 9$.

Thus, the answer is **C**.

22. A bus takes 2 minutes to drive from one stop to the next, and waits 1 minute at each stop to let passengers board. Zia takes 5 minutes to walk from one bus stop to the next. As Zia reaches a bus stop, if the bus is at the previous stop or has already left the previous stop, then she will wait for the bus. Otherwise she will start walking toward the next stop. Suppose the bus and Zia start at the same time toward the library, with the bus 3 stops behind. After how many minutes will Zia board the bus?

- A 17
- B 19
- C 20
- D 21
- E 23

Solution(s):

Since a bus stops takes 2 minutes to drive and 1 minute at every stop, the bus takes 3 minutes at every stop.

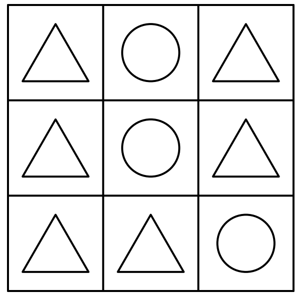
Zia can only stop at every 5 minute interval as that is when she arrives at a bus stop and chooses whether to stop or continue.
Therefore, we can consider the time Zia stops by looking at each of their locations at these intervals.

After 0 minutes, Zia is 3 stops from where the bus started.
After 5 minutes, Zia is 4 stops from where the bus started. Meanwhile, the bus is 2 stops from where it started, waiting. Zia therefore doesn't stop here.
After 10 minutes, Zia is 5 stops from where the bus started. Meanwhile, the bus is between the 3rd and 4th stops from where it started. Zia therefore doesn't stop here.
After 15 minutes, Zia is 6 stops from where the bus started. Meanwhile, the bus is 5 stops from where it started, about to leave. Since the bus is at the previous stop, she will wait here.

It will then take 2 more minutes to get to Zia, so the total time was 17 minutes.

Thus, the answer is **A**.

23. A △ or ◯ is placed in each of the nine squares in a 3×3 grid. Shown below is a sample configuration with three △'s in a line.



How many configurations will have three △'s in a line and three ◯'s in a line?

A 39

B 42

C 78

D 84

E 96

Solution(s):

First, there can't be a horizontal line of one shape and a vertical line of another shape.
This can't happen as it would require a position to take both shapes.

This means we can consider only when the lines are horizontal and only when the lines are vertical.
Since rotating the shapes will take the lines from horizontal to vertical, we only need to check how many vertical lines there are as there are equally as many horizontal lines.

Now, when finding configurations, we can have 3 separate vertical lines or only 2 vertical lines. We can split this into cases.

Case 1: 3 lines: There are 2 choices for the first line (which could be all triangles and all circles), 2 choices for the second line, and 2 choices for the third line. This would make $2 \cdot 2 \cdot 2 = 8$ choices. However, there are two cases to ignore in which each line is the same. These cases would ensure that only one shape has a line. Therefore, with 3 lines, we have $8 - 2 = 6$ total choices.

Case 2: 2 lines: Since there are two lines and we need at least one line of each shape, each shape can only have one line. There are 3 ways to choose a spot for the line of triangles a 2 ways to choose a spot for the line of circles. Now, with the remaining 3 spots, we need to ensure that a line isn't creates or it would fall into the other case. This would mean we have $2 \cdot 2 \cdot 2 - 2 = 6$ choices for the remaining line. Totally, we have $3 \cdot 2 \cdot 6 = 36$ choices with 2 lines.

This means we have $6 + 36 = 42$ configurations with vertical lines. As shown before, we have the same number of horizontal configurations, so we have 42 horizontal configurations. This makes 84 total cases.

Thus, the answer is **D**.

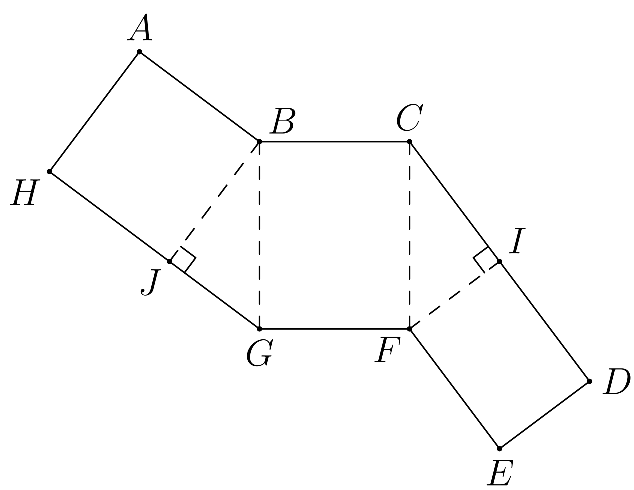
24. The figure below shows a polygon $ABCDEFGH$, consisting of rectangles and right triangles. When cut out and folded on the dotted lines, the polygon forms a triangular prism. Suppose that:

$$AH = EF = 8$$

and

$$GH = 14.$$

What is the volume of the prism?



A 112

B 128

C 192

D 240

E 288

Solution(s):

Since EF maps to GF on the map, we know

$$GF = EF = 8.$$

Since $GFCB$ is a rectangle with GF and BC being opposite, we know

$$BC = GF = 8.$$

Since AB maps to BC on the map, we know

$$AB = BC = 8.$$

Since $HJBA$ is a rectangle with HJ and AB being opposite, we know

$$HJ = AB = 8.$$

Since $HJBA$ is a rectangle with BJ and AJ being opposite, we know

$$BJ = AH = 8.$$

Since $GJ = GH - JH$, we know

$$GJ = 14 - 8 = 6.$$

The area of BJG is $\dfrac{6 \cdot 8}{2} = 24$. Now, we can make this prism complete with bases of BJG and CIF. The volume of a prism is the area of the base times the altitude to the base. We can use BJG as our base and GF as our altitude. This would make our volume equal to

$$[BJG] \cdot GF = 24 \cdot 8 = 192.$$

Thus, the answer is **C**.

25. A cricket randomly hops between 4 leaves, on each turn hopping to one of the other 3 leaves with equal probability. After 4 hops, what is the probability that the cricket has returned to the leaf where it started?

A $\frac{2}{9}$

B $\frac{19}{80}$

C $\frac{20}{81}$

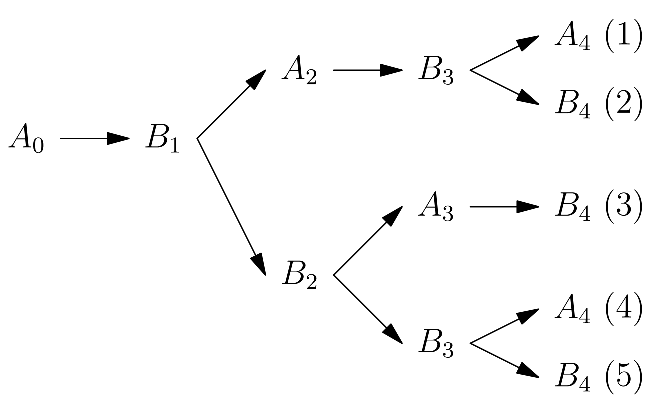
D $\frac{1}{4}$

E $\frac{7}{27}$

Solution(s):

We begin by defining the action of the cricket jumping to the starting leaf as A and the action of it jumping to any of the other three leaves as B. With this in mind, note that when the cricket is on the first leaf, the probability of it jumping to the first leaf, $P(A)$, is zero, and the probability that it jumps to any of the other leaves, $P(B)$, is 1. Similarly, notice that when the cricket is not on the first leaf, $P(A) = \frac{1}{3}$ and $P(B) = \frac{2}{3}$.

Therefore, let us map out all possible paths the cricket can take in four hops, and track the probability of each path as follows:



We want to find the total probability of the cricket landing on the first leaf, and as such, we want to find the total probability that the last node in the diagram is

A_4. This means that we want to find $P(\sum A_4)$:

$$
\begin{aligned}
&= P_{(1)} + P_{(4)} \\
&= \left(1 \cdot \frac{1}{3} \cdot 1 \cdot \frac{1}{3}\right) \\
&+ \left(1 \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3}\right) \\
&= \frac{1}{9} + \frac{4}{27} \\
&= \frac{7}{27}
\end{aligned}
$$

Thus, the answer is **E**.

Problems: https://live.poshenloh.com/past-contests/amc8/2022