2023 AMC 8 Solutions

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https://live.poshenloh.com/past-contests/amc8/2023/solutions

1. What is the value of

$$(8 \times 4 + 2) - (8 + 4 \times 2)?$$

A 0

B 6

C 10

D 18

E 24

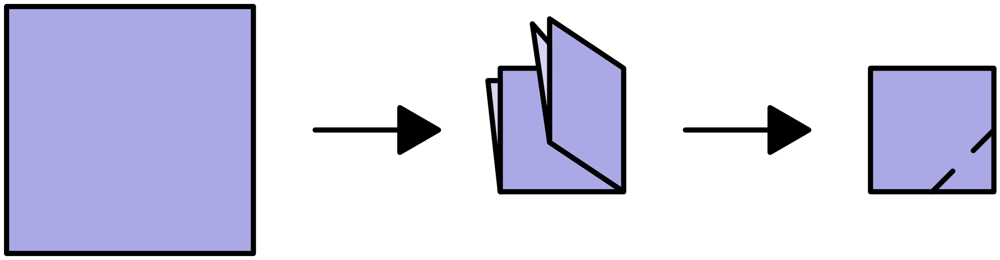
Solution(s):

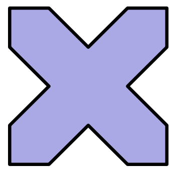
We can simplify this as follows.

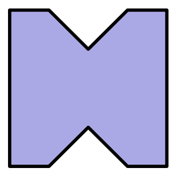
$$\begin{aligned}(32 + 2) - (8 + 8) &= 34 - 16 \\ &= 18\end{aligned}$$

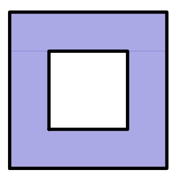
Thus, **D** is the correct answer.

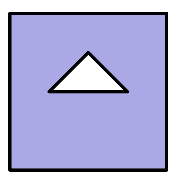
2. A square piece of paper is folded twice into four equal quarters, as shown below, then cut along the dashed line. When unfolded, the paper will match which of the following figures?

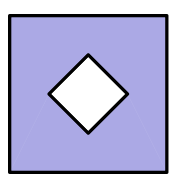


A 

B 

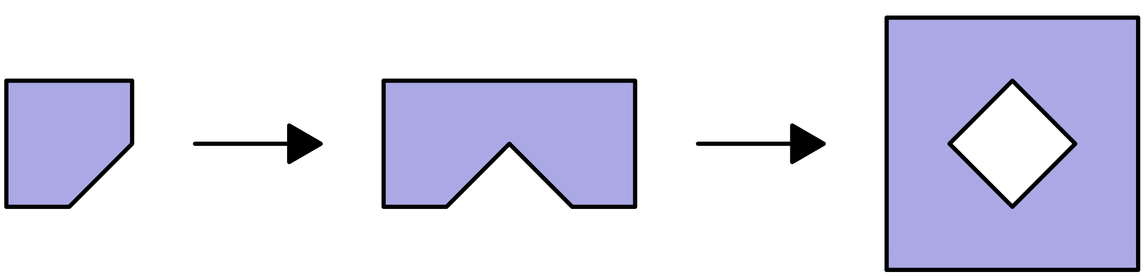
C 

D 

E 

Solution(s):

We can unfold the cut up paper to achieve the following figure.



Thus, **E** is the correct answer.

3. *Wind chill* is a measure of how cold people feel when exposed to wind outside. A good estimate for wind chill can be found using this calculation:

$$W = T - 0.7 \cdot S$$

Where W represents the wind chill, T respresents air temperature measured in degrees Fahrenheit $(^\circ F)$, and S represents wind speed measured in miles per hour (mph).

Suppose the air temperature is $36^\circ F$ and the wind speed is 18 mph. Which of the following is closest to the approximate wind chill?

A 18

B 23

C 28

D 32

E 35

Solution(s):

Using the formula, we can calculate the wind chill to be

$$\begin{aligned} 36 - 0.7 \cdot 18 &= 36 - 12.6 \\ &= 23.4. \end{aligned}$$

Thus, **B** is the correct answer.

4. The numbers from 1 to 49 are arranged in a spiral pattern on a square grid, beginning at the center. The first few numbers have been entered into the grid below. Consider the four numbers that will appear in the shaded squares, on the same diagonal as the number 7. How many of these four numbers are prime?

		5	4	3		
		6	1	2		
		7				

A 0

B 1

C 2

D 3

E 4

Solution(s):

We can fill in the other numbers to get the complete grid.

37	36	35	34	33	32	31
38	17	16	15	14	13	30
39	18	5	4	3	12	29
40	19	6	1	2	11	28
41	20	7	8	9	10	27
42	21	22	23	24	25	26
43	44	45	46	47	48	49

From this, we can see that the only prime numbers in the shaded boxes are $19, 23$, and 47.

Thus, **D** is the correct answer.

5. A lake contains 250 trout, along with a variety of other fish. When a marine biologist catches and releases a sample of 180 fish from the lake, 30 are identified as trout. Assume that the ratio of trout to the total number of fish is the same in both the sample and the lake. How many fish are there in the lake?

A 1250

B 1500

C 1750

D 1800

E 2000

Solution(s):

Note that $\frac{30}{180} = \frac{1}{6}$. This means that a sixth of the fish in the lake are trout, or in other words, the total number of fish is 6 times the number of trout.

Therefore, there are $6 \cdot 250 = 1500$ fish in the lake.

Thus, **B** is the correct answer.

6. The digits $2, 0, 2,$ and 3 are placed in the expression below, one digit per box. What is the maximum possible value of the expression?

$$\square^{\square} \times \square^{\square}$$

- A 0
- B 8
- C 9
- D 16
- E 18

Solution(s):

Note that we do not want 0 as a base, since that would make the expression equal to 0.

This means that 0 must be an exponent. The number whose exponent is 0 will automatically evaluate to 1.

Therefore, we should minimize this number, so we can put 2^0.

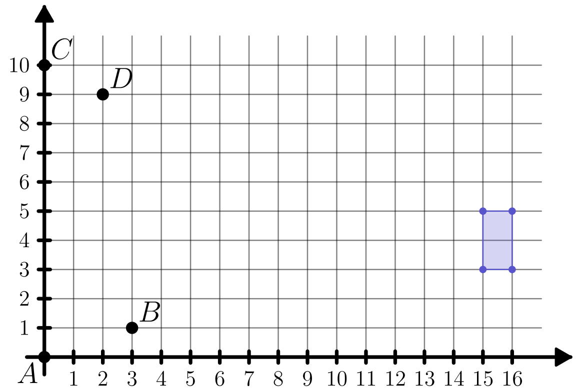
Now the other term is 2^3 or 3^2. Clearly, 3^2 is larger, so we should use that.

This gives us a final value of

$$2^0 \times 3^2 = 1 \times 9 = 9.$$

Thus, **C** is the correct answer.

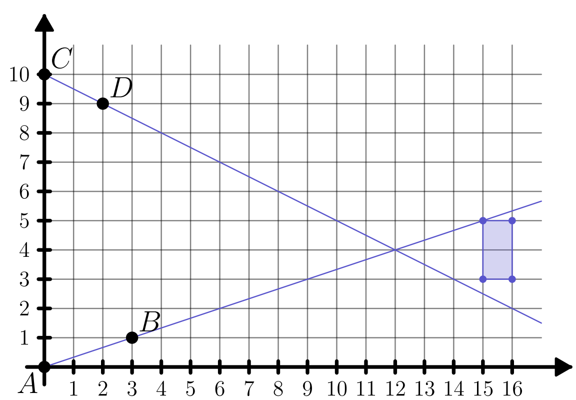
7. A rectangle, with sides parallel to the x-axis and y-axis, has opposite vertices located at $(15, 3)$ and $(16, 5)$. A line is drawn through points $A(0, 0)$ and $B(3, 1)$. Another line is drawn through points $C(0, 10)$ and $D(2, 9)$. How many points on the rectangle lie on at least one of the two lines?



- A 0
- B 1
- C 2
- D 3
- E 4

Solution(s):

We can graph the two lines.



From this, we see that only the top left corner of the rectangle intersects either line.

Thus, **B** is the correct answer.

8. Lola, Lolo, Tiya, and Tiyo participated in a ping pong tournament. Each player competed against each of the other three players exactly twice. Shown below are the win-loss records for the players. The numbers 1 and 0 represent a win or loss, respectively. For example, Lola won five matches and lost the fourth match. What was Tiyo's win-loss record?

Player	Result
Lola	111011
Lolo	101010
Tiya	010100
Tiyo	?

A 000101

B 001001

C 010000

D 010101

E 011000

Solution(s):

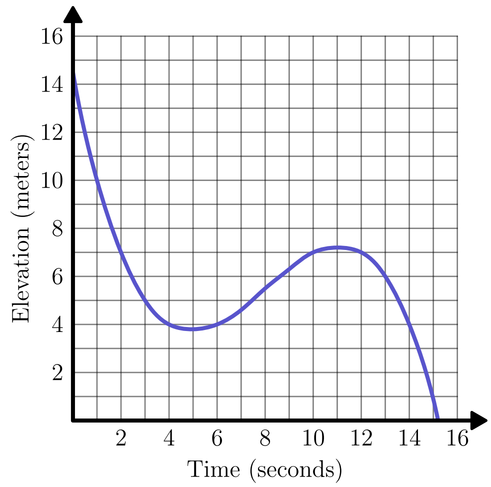
Note that for each round, there are going to 2 winners and 2 losers (two matches each with a winner and loser).

This means that for each match, there should be 2 ones and 2 zeros.

Using this fact, we can deduce that Tiyo's win-loss record is 000101.

Thus, **A** is the correct answer.

9. Malaika is skiing on a mountain. The graph below shows her elevation, in meters, above the base of the mountain as she skis along a trail. In total, how many seconds does she spend at an elevation between 4 and 7 meters?



A 6

B 8

C 10

D 12

E 14

Solution(s):

The first time that she hits an elevation of 7 meters is at 2 seconds.

She then dips below 4 meters after 4 seconds. This adds $4 - 2 = 2$ seconds to the total answer.

Malaika then goes above 4 meters at 6 seconds. She hits 7 meters again at 10 seconds.

This adds $10 - 6 = 4$ more seconds to the total. She finally dips below 7 meters for the last time at 12 seconds.

She then falls below 4 meters at 14 seconds, finally adding $14 - 12 = 2$ seconds to the total time.

The desired answer is therefore

$$2 + 4 + 2 = 8 \text{ seconds.}$$

Thus, **B** is the correct answer.

10. Harold made a plum pie to take on a picnic. He was able to eat only $\frac{1}{4}$ of the pie, and he left the rest for his friends. A moose came by and $\frac{1}{3}$ of what Harold left behind. After that, a porcupine ate $\frac{1}{3}$ of what the moose left behind. How much of the original pie still remained after the porcupine left?

A $\frac{1}{12}$

B $\frac{1}{6}$

C $\frac{1}{4}$

D $\frac{1}{3}$

E $\frac{5}{12}$

Solution(s):

Harold left $1-\frac{1}{4}=\frac{3}{4}$ of the pie for his friends.

The moose ate $\frac{1}{3}\cdot\frac{3}{4}=\frac{1}{4}$ of the pie, leaving $\frac{3}{4}-\frac{1}{4}=\frac{1}{2}$ of the pie.

Finally, the porcupine ate $\frac{1}{3}\cdot\frac{1}{2}=\frac{1}{6}$ of the pie. This leaves $\frac{1}{2}-\frac{1}{6}=\frac{1}{3}$ of the pie.

Thus, **D** is the correct answer.

11. NASA's Perseverance Rover was launched on July $30, 2020$. After traveling $292, 526, 838$ miles, it landed on Mars in Jezero Crater about 6.5 months later. Which of the following is closest to the Rover's average interplanetary speed in miles per hour?

A $6,000$

B $12,000$

C $60,000$

D $120,000$

E $600,000$

Solution(s):

We can round the distance up to $300,000,000$ miles. We can also approximate 6.5 months as

$$30 \times 6.5 = 195 \approx 200$$

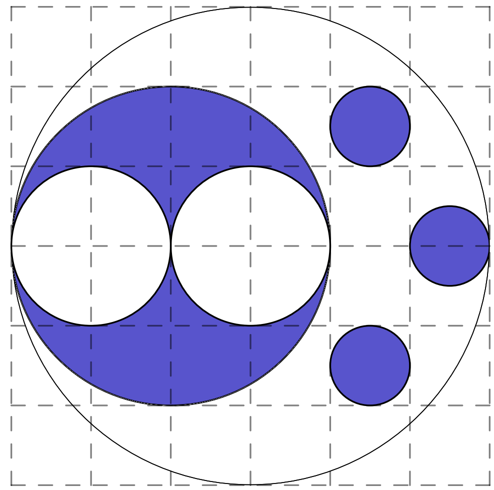
days. This means that the Rover traveled

$$300,000,000 \div 200 = 1,500,000$$

miles per day. Dividing this by 24 to get the speed in miles per hour yields $62,500$, which is close to $60,000$.

Thus, **C** is the correct answer.

12. The figure below shows a large unshaded circle with a number of smaller unshaded and shaded circles in its interior. What fraction of the interior of the large unshaded circle is shaded?



A $\frac{1}{4}$

B $\frac{11}{36}$

C $\frac{1}{3}$

D $\frac{19}{36}$

E $\frac{5}{9}$

Solution(s):

WLOG, assume that each square has a side length of 2 units.

This means that there are 3 shaded unit circles, which total to $3 \cdot 1^2\pi = 3\pi$ area.

There is also a shaded circle with radius 4 with two white circles of radius 2 inside.

This gives us an extra shaded area of

$$\begin{aligned} 4^2\pi - 2 \cdot 2^2\pi &= 16\pi - 8\pi \\ &= 8\pi. \end{aligned}$$

Adding these values together yields

$$8\pi + 3\pi = 11\pi.$$

The area of the large white circle is

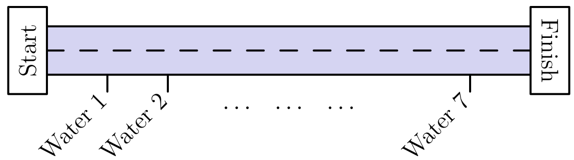
$$6^2\pi = 36\pi.$$

Therefore, the desired fraction is

$$\frac{11\pi}{36\pi} = \frac{11}{36}.$$

Thus, **B** is the correct answer.

13. Along the route of a bicycle race, 7 water stations are evenly spaced between the start and finish lines, as shown in the figure below. There are also 2 repair stations evenly spaced between the start and finish lines. The 3rd water station is located 2 miles after the 1st repair station. How long is the race in miles?



A 8

B 16

C 24

D 48

E 96

Solution(s):

The 3rd water station is located $\frac{3}{8}$ of the way along the race (the water stations split the race up into 8 equal spaces).

The first repair station is located $\frac{1}{3}$ of the way along the race. The distance between the two is

$$\frac{3}{8} - \frac{1}{3} = \frac{1}{24}$$

the length of the race. We know that this equals 2, which means the race is $24 \cdot 2 = 48$ miles long.

Thus, **D** is the correct answer.

14. Nicolas is planning to send a package to his friend Anton, who is a stamp collector. To pay for the postage, Nicolas would like to cover the package with a large number of stamps. Suppose he has a collection of 5-cent, 10-cent, and 25-cent stamps, with exactly 20 of each type. What is the greatest number of stamps Nicolas can use to make exactly $7.10 in postage?

(Note: The amount $7.10 corresponds to 7 dollars and 10 cents. One dollar is worth 100 cents.)

A 45

B 46

C 51

D 54

E 55

Solution(s):

Note that we want to get $7 \cdot 100 + 10 = 710$ cents. Let us try to see if it is possible to use up all of the 5-cent and 10-cent stamps.

All of these two types of stamps combined would be worth

$$20(5 + 10) = 20 \cdot 15 = 300$$

cents. We then would need $710 - 300 = 410$ cents, which cannot be created with just 25-cent stamps.

We can, however, make 425 cents with 25-cent stamps. Using only 19 each of 5 and 10-cent stamps would total

$$300 - 5 - 10 = 285$$

cents. This means we would then need $710 - 285 = 425$ cents. This can be achieved with $425 \div 25 = 17$ 25-cent stamps.

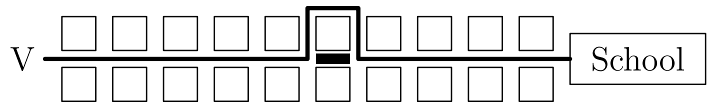
This lets us use

$$\begin{aligned} 2 \cdot 19 + 17 &= 38 + 17 \\ &= 55 \end{aligned}$$

stamps.

Thus, **E** is the correct answer.

15. Visvam walks half a mile to get to school each day. His route consists of 10 city blocks of equal length and he takes one minute to walk each block. Today, after walking 5 blocks, Visvam discovers that he has to make a detour, walking 3 blocks of equal length instead of 1 block to reach the next corner. From the time he starts his detour, at what speed, in miles per hour, must Visvam walk in order to arrive at school at his usual time?



A 4

B 4.2

C 4.5

D 4.8

E 5

Solution(s):

If half a mile is the same as 10 blocks, then one block is $\frac{1}{2} \div 10 = \frac{1}{20}$ miles.

Starting from the detour, Visvam has to walk $7 \cdot \frac{1}{20} = \frac{7}{20}$ miles.

Normally, from this spot Visvam would take 5 minutes to walk to school. Now he has to travel $\frac{7}{20}$ miles in 5 minutes.

Note that 5 minutes is $\frac{5}{60} = \frac{1}{12}$ miles. This means his speed must be

$$\begin{aligned} \frac{\frac{7}{20}}{\frac{1}{12}} &= 12 \cdot \frac{7}{20} \\ &= \frac{21}{5} \\ &= 4.2 \end{aligned}$$

mph.

Thus, **B** is the correct answer.

16. The letters P, Q, and R are entered into a 20×20 table according to the pattern shown below. How many Ps, Qs, and Rs will appear in the completed table.

$$\begin{array}{|c|c|c|c|c|c}
\vdots & \vdots & \vdots & \vdots & \vdots & \iddots \\
\hline
\mathrm{Q} & \mathrm{R} & \mathrm{P} & \mathrm{Q} & \mathrm{R} & \cdots \\
\hline
\mathrm{P} & \mathrm{Q} & \mathrm{R} & \mathrm{P} & \mathrm{Q} & \cdots \\
\hline
\mathrm{R} & \mathrm{P} & \mathrm{Q} & \mathrm{R} & \mathrm{P} & \cdots \\
\hline
\mathrm{Q} & \mathrm{R} & \mathrm{P} & \mathrm{Q} & \mathrm{R} & \cdots \\
\hline
\mathrm{P} & \mathrm{Q} & \mathrm{R} & \mathrm{P} & \mathrm{Q} & \cdots \\
\hline
\end{array}$$

A 132 Ps, 134 Qs, 134 Rs

B 133 Ps, 133 Qs, 134 Rs

C 133 Ps, 134 Qs, 133 Rs

D 134 Ps, 132 Qs, 134 Rs

E 134 Ps, 133 Qs, 133 Rs

Solution(s):

Since $20 = 3 \cdot 6 + 2$, the bottom 2 letters in each column will occur one more time than the third letter.

This means that the third letter in each column will occur 6 times, whereas the other 2 will appear 7 times.

Using the same analysis, we can see that R and P appear in the third row 7 times, whereas Q only appears 6 times.

Therefore, in $20 - 7 = 13$ columns, P and R appear 7 times, for a total of $13 \cdot 7 = 91$ times.

They also appear 6 times in 7 columns, adding $6 \cdot 7 = 42$ appearances to the total. This gives us a total of $91 + 42 = 133$ appearances for P and R.

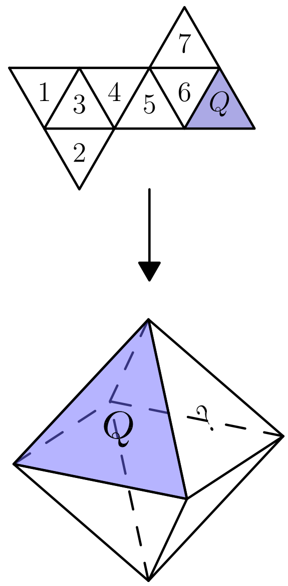
Q will therefore appear

$$\begin{aligned} 20 \cdot 20 - 2 \cdot 133 &= 400 - 266 \\ &= 134 \end{aligned}$$

times.

Thus, **C** is the correct answer.

17. A *regular octahedron* has eight equilateral triangle face with four faces meeting at each vertex. Jun will make the regular octahedron shown in the figure by folding the piece of paper below. Which number face will end up to the right of the shaded region Q?



A 1

B 2

C 3

D 4

E 5

Solution(s):

Begin by observing that when folded, the sides labelled 2,3,4, and 5 form the bottom half of the octahedron. As such, the remaining four faces must make up the top half of the octahedron.

From here, we have narrowed down our possibilities to 1,6, and 7. We can see that 6 will be the number to the left of the shaded region Q. This also gives us that 7 is to the left of 6.

Therefore, we know that the only remaining face, 1, must be to the right of the shaded region Q.

Thus, **A** is the correct answer.

18. Greta Grasshopper sits on a long line of lily pads in a pond. From any lily pad, Greta can jump 5 pads to the right or 3 pads to the left. What is the fewest number of jumps Greta must make to reach the lily pad located 2023 pads to the right of her starting position?

A 405

B 407

C 409

D 411

E 413

Solution(s):

An optimal strategy would be jumping as close as possible with the right jumps and then fine tuning with the left jumps.

It will take at least 404 jumps to get to 2020. Clearly, we cannot get to 2023 in one more jump, so **A** cannot be right.

With 3 jumps, the only way to move forward is with 2 jumps right and the one jump left, but that puts us at 2207.

This shows that **B** cannot happen.

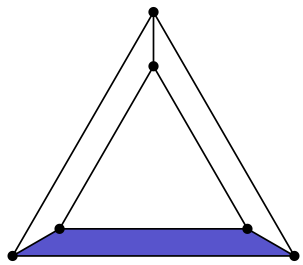
Using 5 jumps, we can jump right twice and jump left thrice, but that puts us at 2021. Jumping right rice and left twice would put us at 2029.

This shows that **C** cannot happen either.

Finally, with 7 jumps, we can jump right thrice and left four times, putting us at 2023.

Thus, **D** is the correct answer.

19. An equilateral triangle is placed inside a larger equilateral triangle so that the region between them can be divided into three congruent trapezoids, as shown below. The side length of the inner triangle is $\frac{2}{3}$ the side length of the larger triangle. What is the ratio of the area of one trapezoid to the area of the inner triangle.



- A $1:3$
- B $3:8$
- C $5:12$
- D $7:16$
- E $4:9$

Solution(s):

Since the inner triangle's side length is $\frac{2}{3}$ the side length of the outer triangle, its area is $\frac{2}{3}^2 = \frac{4}{9}$ the area of the outer triangle.

This means that the three trapezoids are $1 - \frac{4}{9} = \frac{5}{9}$ the area of the outer triangle.

Therefore, one trapezoid is $\frac{5}{9} \div 3 = \frac{5}{27}$ the area of the outer triangle.

This makes the ratio of the areas of one trapezoid and the inner triangle

$$\frac{\frac{5}{27}}{\frac{4}{9}} = \frac{5}{27} \cdot \frac{9}{4} = \frac{5}{12}.$$

Thus, **C** is the correct answer.

20. Two integers are inserted into the list $3, 3, 8, 11, 28$ to double its range. The mode and median remain unchanged. What is the maximum possible sum of the two additional numbers?

A 56

B 57

C 58

D 60

E 61

Solution(s):

Since the median remains unchanged. One number less than 8 and one number greater than 8 is added.

The inequalities are strict since the mode doesn't change, which means that only 3 can appear twice.

The current range is $28 - 3 = 25$. The new range is therefore $2 \cdot 25 = 50$.

To maximize the smaller number, we can set it equal to 7. The larger number is forced to be $50 + 3 = 53$.

Their sum is $53 + 7 = 60$.

Thus, **D** is the correct answer.

21. Alina writes the numbers $1, 2, \cdots, 9$ on separate cards, one number per card. She wishes to divide the cards into 3 groups of 3 cards so that the sum of the numbers in each group will be the same. In how many ways can this be done?

A 0

B 1

C 2

D 3

E 4

Solution(s):

The sum of all the numbers is $\dfrac{9 \cdot 10}{2} = 45$. This means that the sum of each group is $45 \div 3 = 15$.

Consider the group with 9 in it. The other two numbers must add to 6. Therefore, the other cards in this group are 1 and 5 or 2 and 4.

Case 1 : One group is $1, 5, 9$

Consider the group with 8 in it. The other numbers must add to 7. The only option is 3 and 4 with the remaining cards.

The other group is then $2, 6$, and 7. This adds to 15, so this case contributes one possibility.

Case 2 : One group is $2, 4, 9$

Consider the group with 8 in it. As above, the other numbers have to add to 7. The only option is 1 and 6.

The final group is $3, 5$, and 7, which adds to 15. This is another configuration.

We have gone through all the cases, which revealed that there are only 2 possible groupings.

Thus, **C** is the correct answer.

22. In a sequence of positive integers, each term after the second is the product of the previous two terms. The sixth term in the sequence is 4000. What is the first term?

A 1

B 2

C 4

D 5

E 10

Solution(s):

Let x be the first term and y be the second.

Then we get the following sequence.

$$x, y, xy, xy^2, x^2y^3, x^3y^5, \cdots$$

From this, we get that $x^3y^5 = 4000$.

Factoring 4000, we get

$$4000 = 2^5 \cdot 5^3.$$

We need x and y to be integers. The only fifth powers that divides 4000 are 1 and 32.

If $y = 1$, then $x^3 = 4000$, which doesn't work since 4000 is not a perfect cube. Therefore,

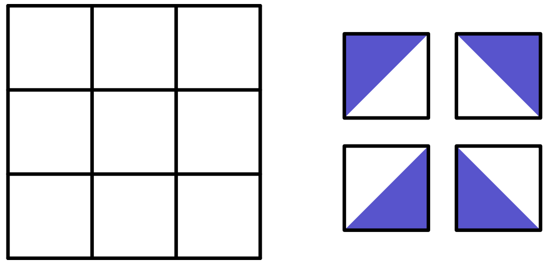
$$y^5 = 32$$

$$y = 2.$$

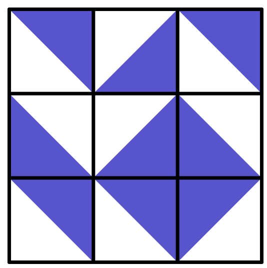
This forces x to equal 5.

Thus, **D** is the correct answer.

23. Each square in a 3×3 grid is randomly filled with one of the 4 gray-and-white tiles shown below on the right.



What is the probability that the tiling will contain a large gray diamond in one of the smaller 2×2 grids? Below is an example of such a tiling.



A $\frac{1}{1024}$

B $\frac{1}{256}$

C $\frac{1}{64}$

D $\frac{1}{16}$

E $\frac{1}{4}$

Solution(s):

There are a total of 4^9 configurations. The 2×2 square that contains the large gray diamond can occur in 4 spots (as there are four 2×2 squares in the big square).

The four tiles in the square have a fixed tile. The other 5 squares can be any tiles, which allows for 4^5 configurations for the other tiles.

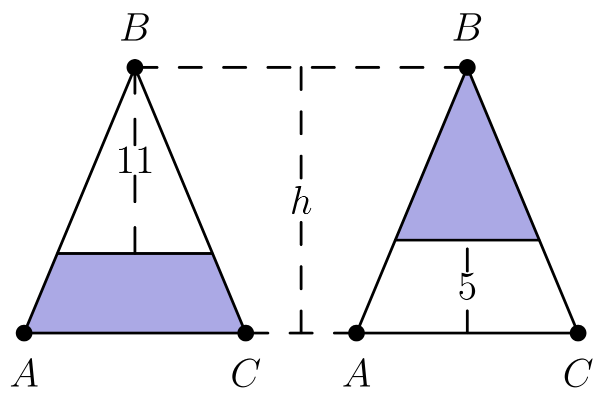
The total number of configurations that include the large gray diamond is therefore $4 \cdot 4^5 = 4^6$.

The desired probability is then

$$\frac{4^6}{4^9} = \frac{1}{4^3} = \frac{1}{64}.$$

Thus, **C** is the correct answer.

24. Isosceles triangle ABC has equal side lengths AB and BC. In the figures below, segments are drawn parallel to $\overline{AC}$ so that the shaded portions of $\triangle ABC$ have the same area. The heights of the two unshaded portions are 11 and 5 units, respectively. What is the height h of $\triangle ABC$?



A 14.6

B 14.8

C 15

D 15.2

E 15.4

Solution(s):

Let a be the area of $\triangle ABC$.

Note that the smaller triangles are similar to $\triangle ABC$. This means that the ratio of their areas is the ratio of their side lengths squared.

Then we get that

$$a - a \cdot \left(\frac{11}{h}\right)^2$$
$$= a \cdot \left(\frac{h-5}{h}\right)^2.$$

The left side is the area of the whole triangle minus the area of the unshaded region of the left triangle.

The right hand side is the area of the shaded triangle. We get this by finding the ratio of their side lengths (which is the same as the ratio of their heights) and squaring it.

Simplifying yields

$$h^2 - 121 = h^2 - 10h + 25.$$

This simplifies to

$$10h = 146$$

$$h = 14.6.$$

Thus, **A** is the correct answer.

25. Fifteen integers $a_1, a_2, a_3, \cdots, a_{15}$ are arranged in order on a number line. The integers are equally spaced and have the property that

$$1 \leq a_1 \leq 10,$$

$$13 \leq a_2 \leq 20,$$

and

$$241 \leq a_{15} \leq 250.$$

What is the sum of the digits of a_{14}?

A 8

B 9

C 10

D 11

E 12

Solution(s):

Let d be the common difference. If we let $a_1 = 10$ and $a_{15} = 241$, we see that

$$d \geq \frac{241 - 10}{14} = 6.5.$$

Since all the numbers are integers, d must be at least 17.

Also, if $a_1 = 1$ and $a_{15} = 250$, we get that

$$d \leq \frac{250 - 1}{14} \approx 17.8.$$

Once again since all the numbers are integers, d is at most 17. This tells us that d is 17.

Note that $17 \cdot 14 = 238$. This means that a_1 must be at least 3 for a_{15} to be within the desired range.

If a_1 is greater than 3, however, a_2 becomes greater than 20, which is not allowed.

Now we know that $a_1 = 3$ and $d = 17$. This tell us that

$$a_{14} = 3 + 13 \cdot 17 = 224.$$

Therefore, sum of the digits is

$$2 + 2 + 4 = 8.$$

Thus, **A** is the correct answer.

Problems: https://live.poshenloh.com/past-contests/amc8/2023