2024 AMC 8 Solutions

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1. What is the ones digit of

$$222,222 - 22,222 - 2,222 - 222 - 22 - 2?$$

A 0

B 2

C 4

D 6

E 8

Solution(s):

We only need to consider the ones digits of each number (except for the first one so we avoid getting a negative answer):

$$22 - 2 - 2 - 2 - 2 - 2 = 12$$

which has a ones digit of 2.

Thus, **B** is the correct answer.

2. What is the value of this expression in decimal form?

$$\frac{44}{11}+\frac{110}{44}+\frac{44}{1100}$$

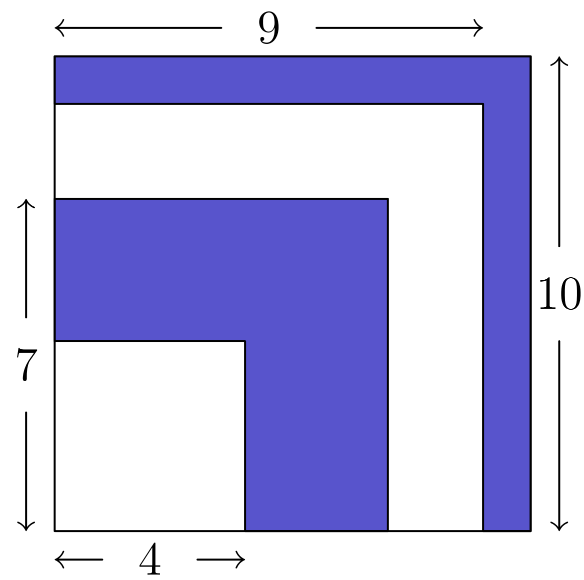
- A 6.4
- B 6.504
- C 6.54
- D 6.9
- E 6.94

Solution(s):

We can simplify the fractions by taking out the common factor 11: $\frac{44}{11}$ simplifies to 4, $\frac{110}{44}$ simplifies to $\frac{10}{4}=\frac{5}{2}=2.5$, and $\frac{44}{1100}$ simplifies to $\frac{4}{100}=0.04$. Therefore, we have $4+2.5+0.04=6.54$.

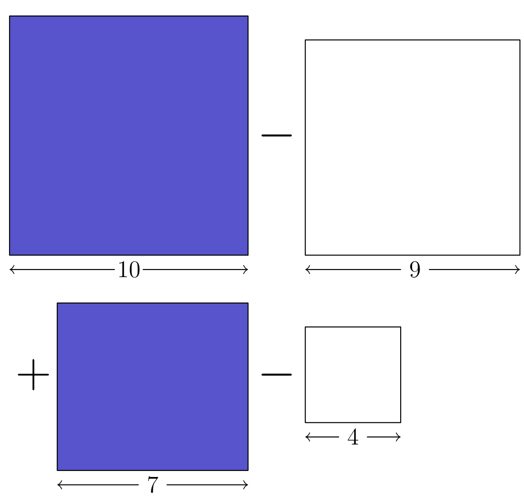
Thus, **C** is the correct answer.

3. Four squares of side length 4, 7, 9, and 10 units are arranged in increasing size order so that their left edges and bottom edges align. The squares alternate in color, as shown in the figure. What is the area of the visible colored region in square units?



- A 42
- B 45
- C 49
- D 50
- E 52

Solution(s):



The visible colored region includes the area of the square with side length 10 subtracted by the area of the square with side length 9 plus the area of the square with side length 7 minus the area of the square with side length 4. Hence, we get $100 - 81 + 49 - 16 = 52$.

Thus, **E** is the correct answer.

4. When Yunji added all the integers from 1 to 9, she mistakenly left out a number. Her incorrect sum turned out to be a square number. Which number did Yunji leave out?

A 5

B 6

C 7

D 8

E 9

Solution(s):

To find the number that Yunji left out, we need to find the sum of the integers from 1 to 9 and find its difference with the largest perfect square below the sum. We can calculate the sum of the integers from 1 to 9 as follows:

$$1 + \ldots + 9 = \frac{9(9+1)}{2} = 45$$

The largest perfect square less than 45 would be 36 and $45 - 36 = 9$.

Thus, **E** is the correct answer.

5. Aaliyah rolls two standard 6-sided dice. She notices that the product of the two numbers rolled is a multiple of 6. Which of the following integers *cannot* be the sum of the two numbers?

A 5

B 6

C 7

D 8

E 9

Solution(s):

can draw a 2x2 chart of possible rolls

We can simply list down all possible combinations of the two rolls:

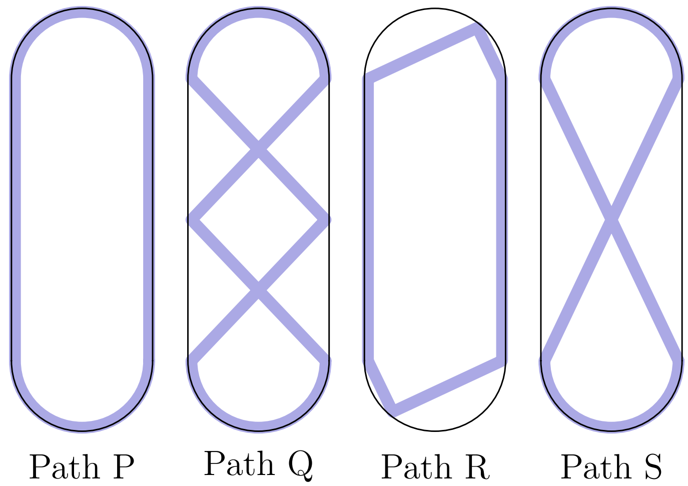
$$(1,6),(2,3),(2,6),(3,6),$$

$$(4,6),(5,6),(6,6)$$

Respectively, the sums would be $7,5,8,9,10,11,12$. Among the choices, only 6 is not a possible sum.

Thus, **B** is the correct answer.

6. Sergei skated around an ice rink, gliding along different paths. The gray lines in the figures below show four of the paths labeled P, Q, R, and S. What is the sorted order of the four paths from shortest to longest?

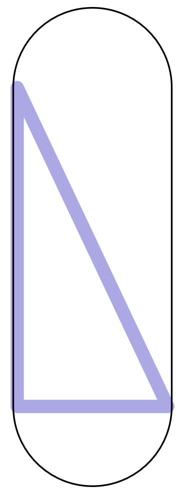


- **A** P, Q, R, S
- **B** P, R, S, Q
- **C** Q, S, P, R
- **D** R, P, S, Q
- **E** R, S, P, Q

Solution(s):

From inspection, we can notice that Path R is the shortest path since it avoids going through the entire loop by taking straight-line shortcuts instead of going through the arc portions of the rink. This leaves us with two possible answer choices, D and E, which only differ in how they sort Path P and S.

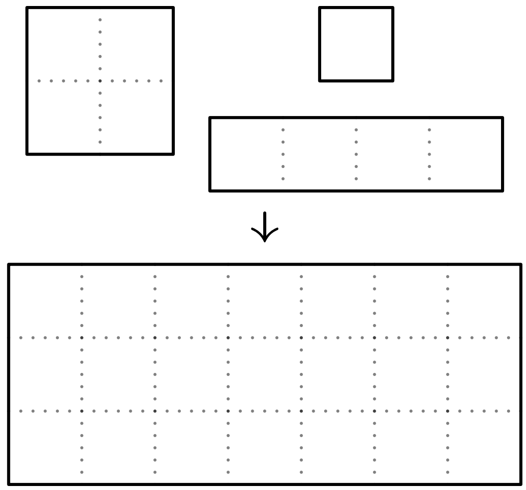
To determine which path is longer between Path P and S, notice that the difference between the two paths can be related by a right triangle as shown here:



The hypotenuse of the triangle was traversed by Sergei in Path S while one of the legs is part of Path P. By the Pythagorean Theorem, we know that the hypotenuse will always be longer than the legs, and so Path S will be longer than Path P.

Thus, **B** is the correct answer.

7. A 3×7 rectangle is covered without overlap by 3 shapes of tiles: 2×2, 1×4, and 1×1, shown below. What is the minimum possible number of 1×1 tiles used?



A 1

B 2

C 3

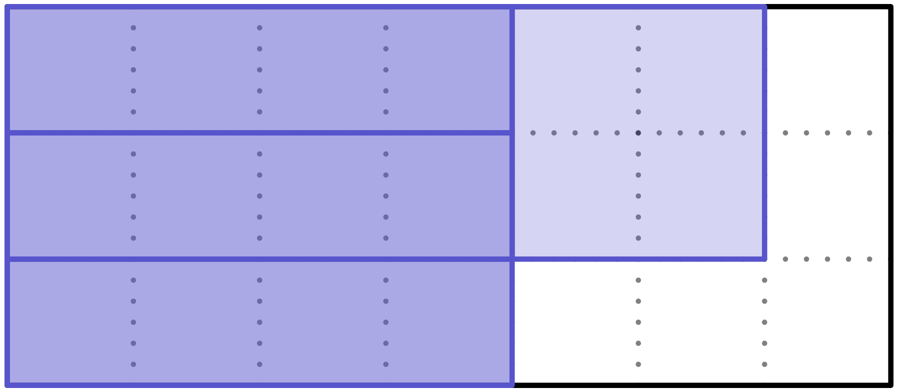
D 4

E 5

Solution(s):

Notice that if we only fill the 3×7 rectangle using 2×2 and 1×4 tiles, then we will always be filling the rectangle in multiples of 4. From the answer choices, it will suffice to consider the cases where we are able to fill 16 or 20 tiles out of the 21 tiles of the rectangle, since the former will leave us 5 spaces for the 1×1 tiles while the latter will leave us with one. The other options are not possible since those numbers of tiles cannot be arrived by subtracting any multiple of 4 from 21.

By attempting to place the 2×2 and 1×4 tiles, we can immediately notice that it's not possible to fill 20 tiles in the rectangle, and we can easily find cases where we are left with five tiles.



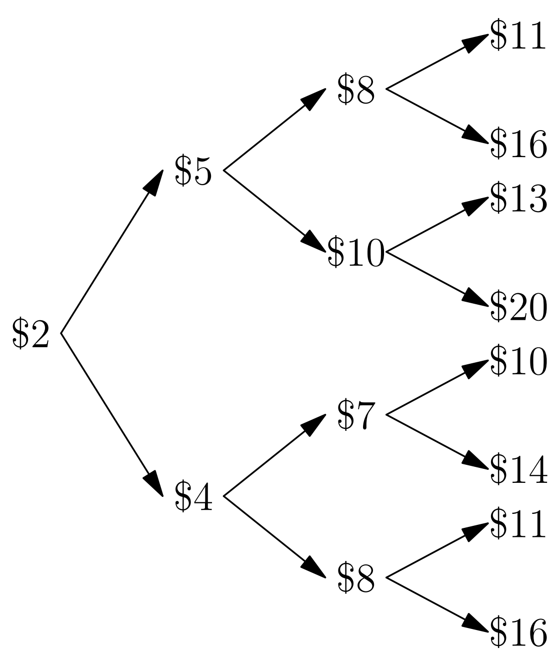
Thus, **E** is the correct answer.

8. On Monday Taye has \$2. Every day, he either gains \$3 or doubles the amount of money he had on the previous day. How many different dollar amounts could Taye have on Thursday, 3 days later?

- A 3
- B 4
- C 5
- D 6
- E 7

Solution(s):

We can systematically exhaust the cases using a tree diagram where the upper branch covers the case where the amount gains \$3 while the other branch represents the case where the amount doubles.



From the diagram, we can see that we end up with six unique dollar amounts after three days.

Thus, **D** is the correct answer.

9. All of the marbles in Maria's collection are red, green, or blue. Maria has half as many red marbles as green marbles and twice as many blue marbles as green marbles. Which of the following could be the total number of marbles in Maria's collection?

A 24

B 25

C 26

D 27

E 28

Solution(s):

We can let r be the number of red marbles that Maria has. Since Maria has half as many red marbles as green, then we know that she has $2r$ green marbles. Moreover, since she has twice as many blue marbles as green, then she will have $2(2r) = 4r$ blue marbles. Adding these together gives us $r + 2r + 4r = 7r$, and so the answer must be a multiple of 7. Among the answer choices, only 28 is a multiple of 7.

Thus, **E** is the correct answer.

10. In January 1980 the Mauna Loa Observatory recorded carbon dioxide CO_2 levels of 338 ppm (parts per million). Over the years the average CO_2 reading has increased by about 1.515 ppm each year. What is the expected CO_2 level in ppm in January 2030? Round your answer to the nearest integer.

A 399

B 414

C 420

D 444

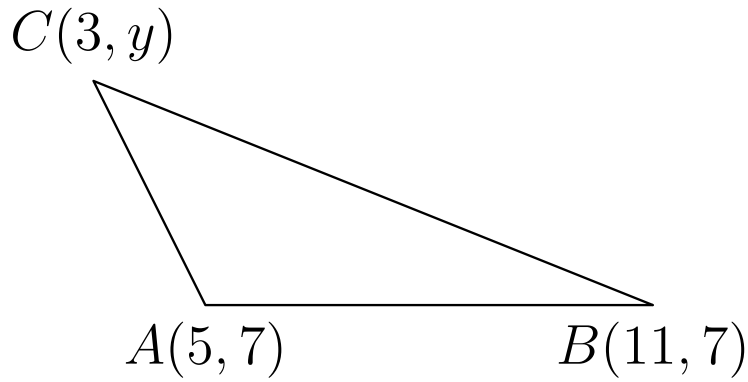
E 459

Solution(s):

There are 50 years between 1980 and 2030, so we can expect the CO_2 reading to increase by $50 \times 1.515 = 75.75 \approx 76$ ppm by 2030. Since the CO_2 reading in 1980 was 338 ppm, then we will have $338 + 76 = 414$ ppm by 2030.

Thus, **B** is the correct answer.

11. The coordinates of $\triangle ABC$ are $A(5,7)$, $B(11,7)$, and $C(3,y)$, with $y > 7$. The area of $\triangle ABC$ is 12. What is the value of y?



A 8

B 9

C 10

D 11

E 12

Solution(s):

Consider the base of the triangle to be $\overline{\text{AB}}$ which has length $11 - 5 = 6$. Given that the area of the triangle is 12, its height must be of length $\dfrac{2(12)}{6} = 4$. Since $y > 7$, then y must be $7 + 4 = 11$.

Thus, **D** is the correct answer.

12. Rohan keeps a total of 90 guppies in 4 fish tanks.

- There is 1 more guppy in the 2nd tank than in the 1st tank.
- There are 2 more guppies in the 3rd tank than in the 2nd tank.
- There are 3 more guppies in the 4th tank than in the 3rd tank.

How many guppies are in the 4th tank?

A 20

B 21

C 23

D 24

E 26

Solution(s):

Let x be the number of guppies in the 1st tank. Hence, there are $x+1$ guppies in the 2nd tank, $x+3$ guppies in the 3rd tank, and $x+6$ guppies in the 4th tank. We then use the fact that there are a total of 90 guppies in the 4 tanks to find x:

$$x+x+1+x+3+x+6=90$$

$$4x+10=90$$

$$x=20.$$

Note that we are **not** yet done since we are asked for the number of guppies in the **4th** tank and not the 1st. There are $x+6=20+6=26$ guppies in the 4th tank.

Thus, **E** is the correct answer.

13. Buzz Bunny is hopping up and down a set of stairs, one step at a time. In how many ways can Buzz start on the ground, make a sequence of 6 hops, and end up back on the ground? (For example, one sequence of hops is up-up-down-down-up-down.)

A 4

B 5

C 6

D 8

E 12

Solution(s):

We can deduce from the choices that it is possible to exhaust all possible cases for this problem. Note that all sequences must start with up (U) and end with down (D), and that it should not be possible to go down more times than Buzz has gone up so far. Keeping this in mind, we can arrive at the following possible cases:

$$UUUDDD$$

$$UUDUDD$$

$$UUDDUD$$

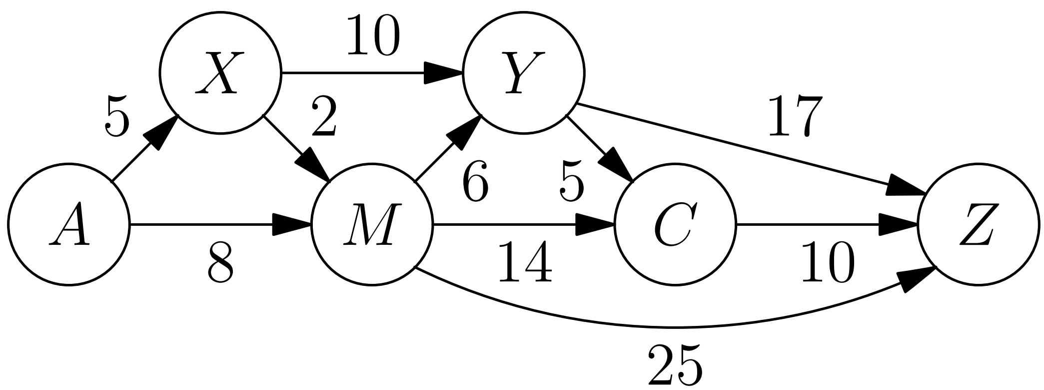
$$UDUDUD$$

$$UDUUDD$$

which is a total of five possible sequences.

Thus, **B** is the correct answer.

14. The one-way routes connecting towns A, M, C, X, Y, and Z are shown in the figure below (not drawn to scale). The distances in kilometers along each route are marked. Traveling along these routes, what is the shortest distance from A to Z in kilometers?



A 28

B 29

C 30

D 31

E 32

Solution(s):

A systematic way of tracking the shortest overall distance to Z is to consider the shortest distance to get to each town from A. For instance, the shortest distance to get to town X from A is 5 km, trivially.

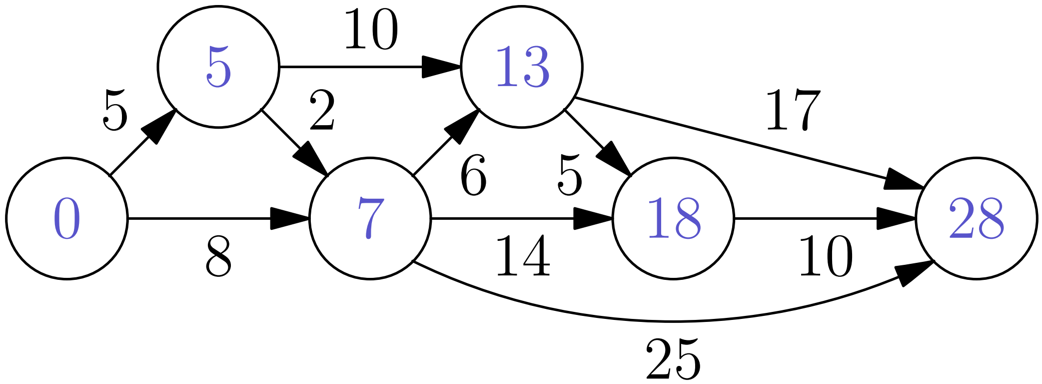
Then, for town M, going to town X first will be shorter compared to going directly from A, so the shortest path to town M has a length of 7 km.

For town Y, it will take us 15 km if we come from town X and only 13 km coming from M, so 13 km is the length of shortest path to Y from A.

Doing the same for town C will give us 18 km as the shortest distance by coming from town Y.

Finally, for town Z, we can either come from town Y, C, or M. The total distance if we come from each three towns respectively would be 30, 28, and 32. Hence, 28 km is the shortest distance from A to Z.

The diagram below summarizes our process. Here, the town name is replaced by the shortest distance to get to that town from town A.



Thus, **A** is the correct answer.

15. Let the letters F, L, Y, B, U, G represent distinct digits. Suppose FLYFLY is the greatest number that satisfies the equation

$$8 \cdot \mathrm{FLYFLY} = \mathrm{BUGBUG}.$$

What is the value of $\mathrm{FLY} + \mathrm{BUG}$?

A 1089

B 1098

C 1107

D 1116

E 1125

Solution(s):

Firstly, note that

$$\mathrm{FLYFLY} = 1001(\mathrm{FLY})$$

and, similarly,

$$\mathrm{BUGBUG} = 1001(\mathrm{BUG})$$

so the equation can be simplified to $8 \cdot \mathrm{FLY} = \mathrm{BUG}$.

For BUG to remain three digits, F must be 1. Moreover, L must also be less than 3 to avoid carrying over 2 to the hundreds digit and making the product 4 digits. Since we need FLY to be the greatest number, L must be 2.

To identify the possible values for Y, we note that so far we have $8(12) = 96$, so we must avoid carrying 4 to the tens digit to keep the resulting product three digits. Hence, $Y \leq 5$. We can try 4 and verify that the resulting product has unique digits that haven't been used yet: $8(124) = 992$, which does not have unique digits. Trying 3, we get $8(123) = 984$, which satisfies our criteria.

Hence,

$$\mathrm{FLY} + \mathrm{BUG} = 123 + 984 = 1107.$$

Thus, **C** is the correct answer.

16. Minh enters the numbers 1 through 81 into the cells of a 9×9 grid in some order. She calculates the product of the numbers in each row and column. What is the least number of rows and columns that could have a product divisible by 3?

A 8

B 9

C 10

D 11

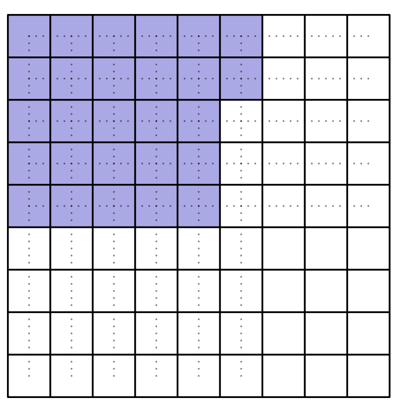
E 12

Solution(s):

We first need to note that there are 27 multiples of 3 from 1 through 81. If any row or column has a multiple of 3, then the product of the numbers in the row or column will be divisible by 3. From this, it is evident that we must try to place all 27 multiples of 3 together in some corner of the grid so the least number of products will be divisible by 3.

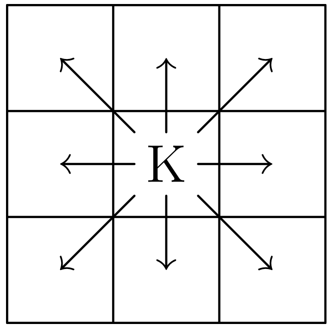
We can place 25 multiples of 3 in a 5×5 grid so only 5 rows and 5 columns will have products that are divisible by 3. The remaining 2 multiples of 3 can then be placed in the 6th column along the 1st and 2nd row of the 5×5 grid so in total we will only have 6 columns and 5 rows that have products that are divisible by 3, for a total of $6 + 5 = 11$ products divisible by 3.

The diagram below illustrates our process. The highlighted cells contain the multiples of 3 while the dotted lines mark the rows or columns that have products which are divisible by 3.



Thus, **D** is the correct answer.

17. A chess king is said to *attack* all the squares one step away from it, horizontally, vertically, or diagonally. For instance, a king on the center square of a 3×3 grid attacks all 8 other squares, as shown below. Suppose a white king and a black king are placed on different squares of a 3×3 grid so that they do not attack each other. In how many ways can this be done?



A 20

B 24

C 27

D 28

E 32

Solution(s):

Firstly, we note that a king cannot be on the center square as it will attack any other piece on the 3×3 grid. To solve this problem, we can simply consider some possible cases: one where a king is in the corner and another when one king is on an edge (but not a corner).

When one king is placed in any corner, then the other king can be placed in 5 other squares without them attacking each other.

K_1		K_2
		K_2
K_2	K_2	K_2

Since there are four corners, we have $5 \times 4 = 20$ possibilities for this case.

Another possible case is when one king is on an edge that is not a corner.

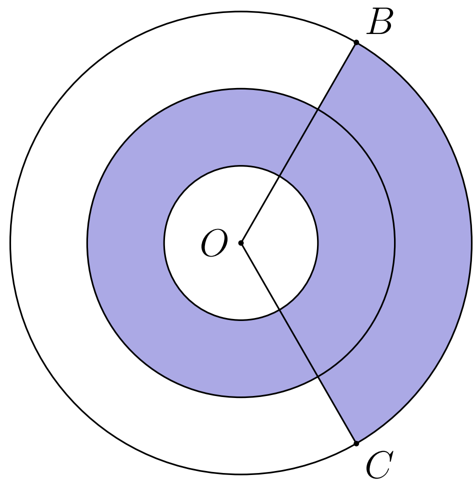
		K_2
K_1		K_2
		K_2

In this scenario, the other king can be in 3 other squares without the two kings attacking each other. There are 4 edges in the grid so we have $4(3) = 12$ possibilities for this case.

Adding the number of possibilities, we get $20 + 12 = 32$ total number of ways.

Thus, **E** is the correct answer.

18. Three concentric circles centered at O have radii of 1, 2, and 3. Points B and C lie on the largest circle. The region between the two smaller circles is shaded, as is the portion of the region between the two larger circles bounded by central angle BOC, as shown in the figure below. Suppose the shaded and unshaded regions are equal in area. What is the measure of $\angle BOC$ in degrees?



A 108

B 120

C 135

D 144

E 150

Solution(s):

Let θ be the measure of $\angle BOC$.

One component of the shaded region is the area of the circle with radius 2 minus the area of the circle with radius 1. This part has area $4\pi - \pi = 3\pi$. The remaining area is a sector of the biggest circle minus the area of the circle with radius 2. This has area $\dfrac{\theta}{360}(9\pi - 4\pi) = \dfrac{\theta}{360}(5\pi)$. Hence, the total area of the shaded region is $3\pi + \dfrac{\theta}{360}(5\pi)$.

Next, we note that the unshaded region is composed of the smallest circle and the unshaded portion of the outer ring. This will have a total area of $\pi + \dfrac{360 - \theta}{360}(5\pi)$

Lastly, we equate the area of both regions and solve for θ :

$$3\pi+\frac{\theta}{360}(5\pi)=\pi+\frac{360-\theta}{360}(5\pi)$$

$$2\pi=\frac{360-\theta-\theta}{360}(5\pi)$$

$$\frac{2}{5}=1-\frac{2\theta}{360}$$

$$2\theta=\frac{3}{5}(360)$$

$$\theta=108.$$

Thus, **A** is the correct answer.

19. Jordan owns 15 pairs of sneakers. Three fifths of the pairs are red and the rest are white. Two thirds of the pairs are high-top and the rest are low-top. The red high-top sneakers make up a fraction of the collection. What is the least possible value of this fraction?

A 0

B $\frac{1}{5}$

C $\frac{4}{15}$

D $\frac{1}{3}$

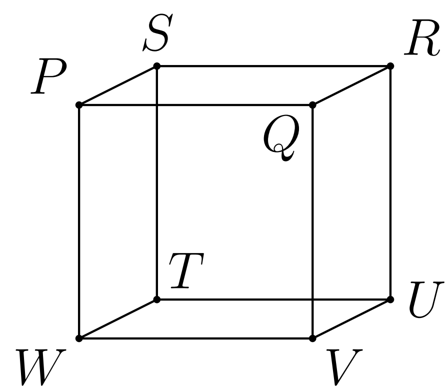
E $\frac{2}{5}$

Solution(s):

Jordan has $\frac{3}{5} \times 15 = 9$ pairs of red sneakers and 6 pairs of white sneakers. Moreover, $\frac{2}{3} \times 15 = 10$ are high-top and 5 are low-top. If we want to minimize the number of red high-top sneakers, then we can set all 6 white sneakers to be high-top, leaving $10 - 6 = 4$ red sneakers as high-top. Hence, the fraction of red high-top sneakers would be $\frac{4}{15}$.

Thus, **C** is the correct answer.

20. Any three vertices of the cube $PQRSTUVW$, shown in the figure below, can be connected to form a triangle. (For example, vertices P, Q, and R can be connected to form isosceles $\triangle PQR$.) How many of these triangles are equilateral and contain P as a vertex?



A 0

B 1

C 2

D 3

E 6

Solution(s):

We first note that we can only form equilateral triangles if we go through the diagonals of the square faces, otherwise at least one angle of the triangle will be different. Afterwards, it is easy to exhaust all possible equilateral triangles that can be formed: $\triangle PVT$, $\triangle PRT$, and $\triangle PRV$.

Thus, **D** is the correct answer.

21. A group of frogs (called an *army*) is living in a tree. A frog turns green when in the shade and turns yellow when in the sun. Initially, the ratio of green to yellow frogs was $3:1$. Then 3 green frogs moved to the sunny side and 5 yellow frogs moved to the shady side. Now the ratio is $4:1$. What is the difference between the number of green frogs and yellow frogs now?

A 10

B 12

C 16

D 20

E 24

Solution(s):

We can let g be the number of green frogs and y be the number of yellow frogs. Initially, we have $g = 3y$. Then, after some frogs moved, we have the following proportion:

$$\frac{g+5-3}{y+3-5} = \frac{4}{1}.$$

Substituting $3y$ for g will allow us to determine the number of yellow frogs originally (y):

$$\frac{3y+2}{y-2} = 4$$

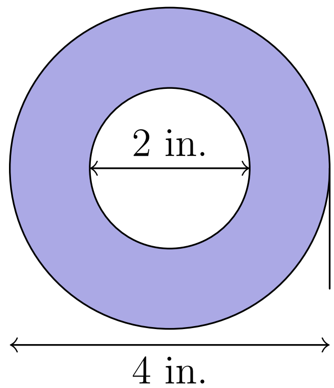
$$3y+2 = 4(y-2) = 4y-8$$

$$y = 10$$

Hence, there were 10 yellow frogs and $3 \times 10 = 30$ green frogs initially. After some frogs moved, we now have $10+3-5 = 8$ yellow frogs and $30+5-3 = 32$ green frogs, giving us a difference of $32-8 = 24$ between the number of green and yellow frogs.

Thus, **E** is the correct answer.

22. A roll of tape is 4 inches in diameter and is wrapped around a ring that is 2 inches in diameter. A cross section of the tape is shown in the figure below. The tape is 0.015 inches thick. If the tape is completely unrolled, approximately how long would it be? Round your answer to the nearest 100 inches.



A 300

B 600

C 1200

D 1500

E 1800

Solution(s):

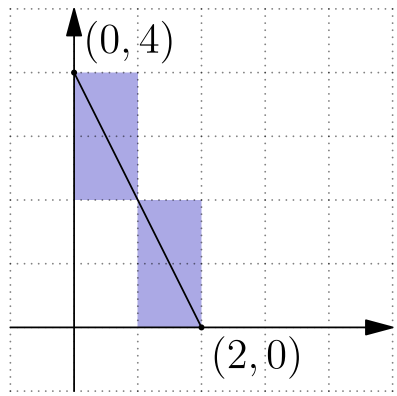
We have a huge margin of error for this problem so we can freely estimate values. Firstly, we note that the entire roll of tape is 1-inch thick, and given that a single tape is 0.015 inches thick, then there are $\frac{1}{0.015} = \frac{2}{0.03} = \frac{200}{3}$ layers of tape in the entire roll.

Then, we must identify an estimate for the circumference of one layer of tape. Near the center, one layer of tape will have a circumference of 2π while the layers near the outer section will have a circumference of about 4π. A good estimate would be to take the average circumference which is 3π. With this, we can estimate the total length for the entire roll:

$$\frac{200}{3}(3\pi) = 200\pi \approx 200(3) \approx 600.$$

Thus, **B** is the correct answer.

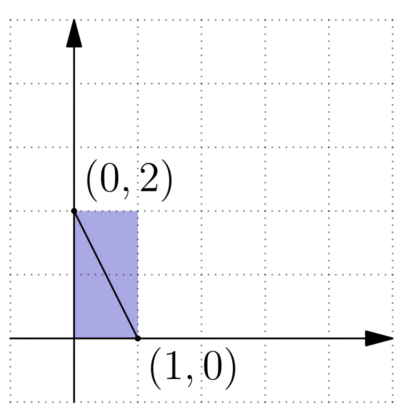
23. Rodrigo has a very large piece of graph paper. First he draws a line segment connecting point $(0,4)$ to point $(2,0)$ and colors the 4 cells whose interiors intersect the segment, as shown below. Next, Rodrigo draws a line segment connecting point $(2000,3000)$ to point $(5000,8000)$. Again he colors the cells whose interiors intersect the segment. How many cells will he color this time?



- A 6000
- B 6500
- C 7000
- D 7500
- E 8000

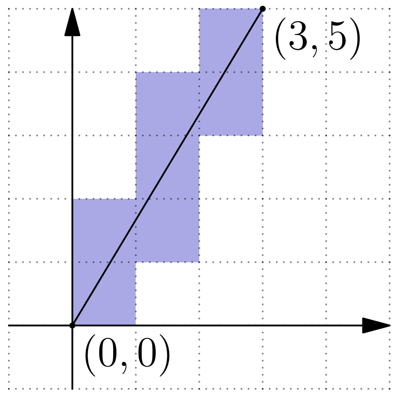
Solution(s):

From the given example, observe that if we consider a different line segment with the same slope, for instance the line connecting points $(0,2)$ and $(1,0)$, then the number of colored cells will be halved.



In general, it is possible to scale down the problem as long as we still have the same slope for the line.

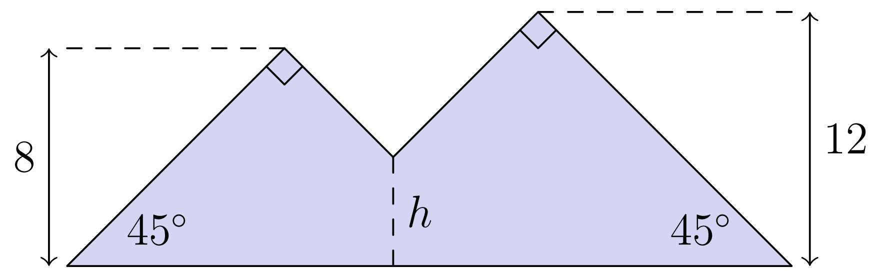
Next, note that the line segment passing through points $(2000, 3000)$ and $(5000, 8000)$ has a slope of $\frac{5}{3}$. Hence, a scaled down version that we can consider is a segment connecting the points $(0, 0)$ and $(3, 5)$.



From the diagram, it is evident that the line passes through 7 cells. We know that this will happen 1000 times as the segment passes through points $(2000, 3000)$ and $(5000, 8000)$. Hence, Rodrigo will need to color $7(1000) = 7000$ cells.

Thus, **C** is the correct answer.

24. Jean made a piece of stained glass art in the shape of two mountains, as shown in the figure below. One mountain peak is 8 feet high and the other peak is 12 feet high. Each peak forms a 90° angle, and the straight sides of the mountains form 45° angles with the ground. The artwork has an area of 183 square feet. The sides of the mountains meet at an intersection point near the center of the artwork, h feet above the ground. What is the value of h?



A 4

B 5

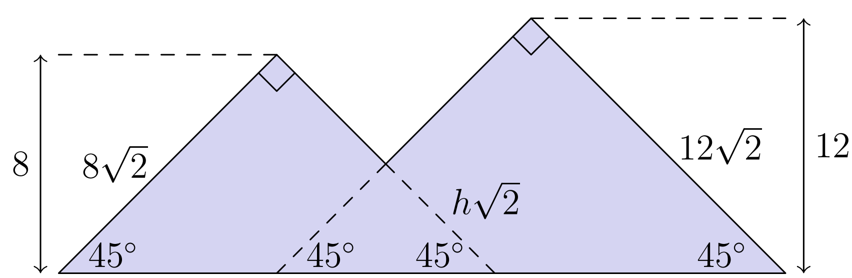
C $4\sqrt{2}$

D 6

E $5\sqrt{2}$

Solution(s):

Observe that the we can extend the two mountains at the intersection point to make three triangles as shown in the diagram below.



All triangles formed are 45°-45°-90° triangles and so based on the given height information, we can determine that the side lengths of the triangles, from left to right, are $8\sqrt{2}, h\sqrt{2}$, and $12\sqrt{2}$. Hence, we have areas of $64, h^2$, and 144, respectively.

To find h, we use the total area of the diagram which we can obtain by adding the area of the two large triangles and subtracting the area of the triangle with height h since this area was counted twice. Doing this, we get

$$64 + 144 - h^2 = 183$$

$$h^2 = 144 + 64 - 183 = 25$$

$$h = 5$$

Thus, **B** is the correct answer.

25. A small airplane has 4 rows of seats with 3 seats in each row. Eight passengers have boarded the plane and are distributed randomly among the seats. A married couple is next to board. What is the probability there will be 2 adjacent seats in the same row for the couple?

A $\frac{8}{15}$

B $\frac{32}{55}$

C $\frac{20}{33}$

D $\frac{34}{55}$

E $\frac{8}{11}$

Solution(s):

For simplicity, we can disregard the order in which passengers are seated so we only consider the number of ways that the 12 seats can be filled by 8 passengers for the total number of possibilities, which is given by $\binom{12}{8} = 495$.

Out of these 495 possibilities, we consider the number of ways where no adjacent seats are available, then subtract this from 495. This scenario can happen when two passengers are occupying the edge seats of one row or one passenger is seated in the middle seat of a row. Hence, we consider the following cases:

• No passenger is in the middle seat (all 8 passengers are on the edge seats): $\binom{8}{8} = 1$ way.

• Exactly 1 passenger is in the middle seat (7 are seated on the edge seats): There are $\binom{4}{1} = 4$ ways where one row can contain a passenger in the middle seat and $\binom{2}{1} = 2$ ways for the eighth passenger to be seated on the row where the one passenger is sat on the middle seat. Hence, the total for this case is $4(2) = 8$ ways.

• Exactly 2 rows have a passenger in the middle seat: $\binom{4}{2} = 6$ ways to select rows with occupied middle seat and another $\binom{4}{2} = 6$ ways for the remaining 2 passengers to be seated in the rows with occupied middle seats. Thus, this case has a total of $6(6) = 36$ possibilities.

• Exactly 3 rows have occupied middle seats: $\binom{4}{3} = 4$ ways to select rows with occupied middle seats and $\binom{6}{3} = 20$ ways for the remaining 3 passengers to be seated on the rows with occupied middle seats, for a total of $4(20) = 80$ ways.

• All 4 rows have occupied middle seats: $\binom{8}{4} = 70$ ways for the remaining 4 passengers to be seated.

Adding all the possibilities in each case, we get $1 + 8 + 36 + 80 + 70 = 195$ ways for there to be no two adjacent seats available. Hence, the probability that the couple will be seated together would be $\dfrac{495 - 195}{495} = \dfrac{20}{33}$.

Thus, **C** is the correct answer.

Problems: https://live.poshenloh.com/past-contests/amc8/2024