2025 AMC 8 Solutions

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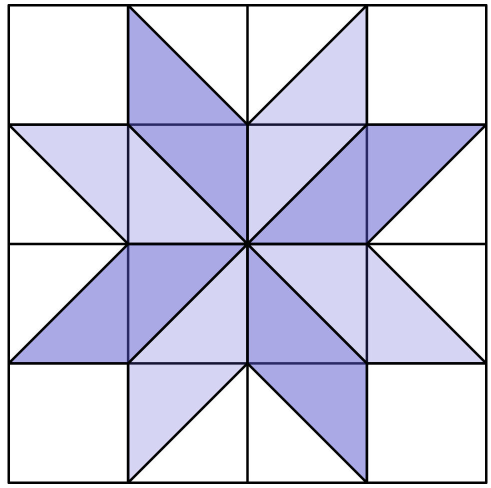
https://live.poshenloh.com/past-contests/amc8/2025/solutions

1. The eight-pointed star, shown in the figure below, is a popular quilting pattern. What percent of the entire 4-by-4 grid is covered by the star?



A 40

B 50

C 60

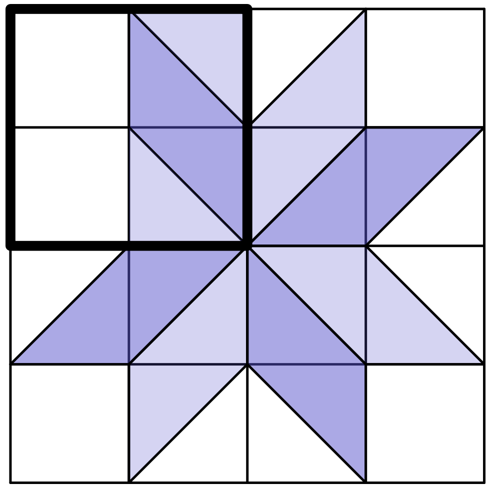
D 75

E 80

Solution:

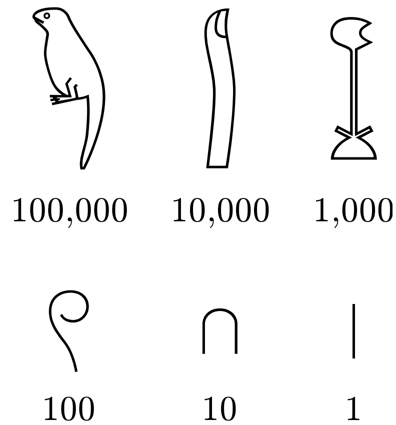
There is symmetry, and so whatever fraction of the top-left quarter is shaded, is the same as the fraction of the entire square that is shaded.

Focus on the top-left quarter. Consider moving a single triangle. The shaded area in the problem is exactly the same as the shaded area in this diagram (and the top-left quarter is outlined in bold):



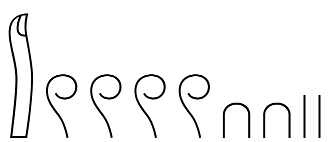
But then it is obvious that exactly half of the top-left quarter is shaded, and so the answer is 50%, which is choice **B**.

2. The table below shows the ancient Egyptian heiroglyphs that were used to represent different numbers.



For example, the number 32 was represented by:

What number was represented by the following combination of heiroglyphs?



- A $1,423$
- B $10,423$
- C $14,023$
- D $14,203$
- E $14,230$

Solution:

We just need to count how many of each type of glyph there are.

There is 1 glyph worth $10,000$.

There are 4 glyphs worth 100 each.

There are 2 glyphs worth 10 each.

There are 3 glyphs worth 1 each.

So, the answer is $10,423$, which is choice **B**.

3. *Buffalo Shuffle-o* is a card game in which all the cards are distributed evenly among all players at the start of the game. When Annika and 3 of her friends play *Buffalo Shuffle-o*, each player is dealt 15 cards. Suppose 2 more friends join the next game. How many cards will be dealt to each player?

A 8

B 9

C 10

D 11

E 12

Solution:

At the start, there are 4 total people (Annika plus 3 friends), each with 15 cards, so there are $4 \times 15 = 60$ cards in total. If 2 more friends join, there will be 6 people in total, and so each should get $60 \div 6 = 10$ cards, which is choice **C**.

4. Lucius is counting backward by 7s. His first three numbers are 100, 93, and 86. What is his 10th number?

A 30

B 37

C 42

D 44

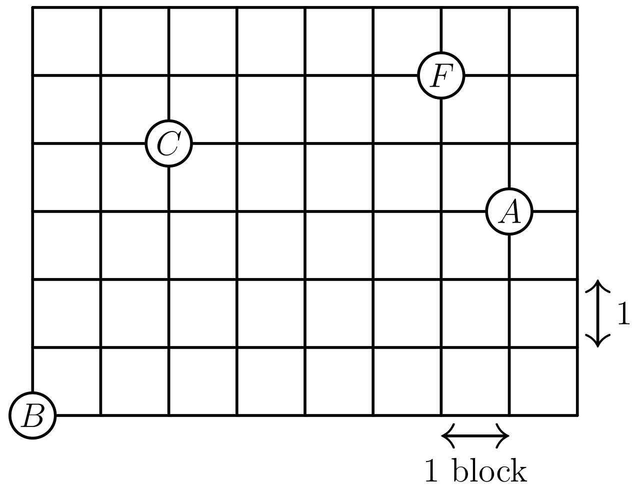
E 47

Solution:

There will be $10 - 1 = 9$ gaps between his first number 100 and his 10th number. Each gap has size 7.

So, he will subtract a total of $9 \times 7 = 63$ from 100, leaving an answer of $100 - 63 = 37$, which is choice **B**.

5. Betty drives a truck to deliver packages in a neighborhood whose street map is shown below. Betty starts at the factory (labeled F) and drives to location A, then B, then C, before returning to F. What is the shortest distance, in blocks, she can drive to complete the route?



A 20

B 22

C 24

D 26

E 28

Solution:

The key idea is that if driving from coordinates (x_1, y_1) to (x_2, y_2), then the shortest distance is $|x_2 - x_1| + |y_2 - y_1|$. This is often called the *Manhattan* distance. It is also equal to the number of horizontal blocks between the locations, plus the number of vertical blocks between the locations.

The shortest distance from F to A is then $1 + 2 = 3$.

The shortest distance from A to B is $7 + 3 = 10$.

The shortest distance from B to C is $2 + 4 = 6$.

The shortest distance from C to F is $4 + 1 = 5$.

Adding up all of these numbers, we get $3 + 10 + 6 + 5 = 24$, which is choice **C**.

One small possible shortcut for the solution is to notice that when going from B to C to F, the visit to C is conveniently along a shortest path from B to F anyway, so we can even remove the requirement to stop at C from the problem.

6. Sekou writes the numbers $15, 16, 17, 18, 19$. After he erases one of the numbers, the sum of the remaining four numbers is a multiple of 4. Which number did he erase?

A 15

B 16

C 17

D 18

E 19

Solution:

The remainders of the five numbers after dividing by 4 are: $3, 0, 1, 2$, and 3. So, the sum of all five numbers modulo 4 is the same as $3+0+1+2+3=9$ which has remainder 1 modulo 4. Therefore, in order to erase a single number and get a sum that is 0 modulo 4, we must erase the number which was 1 modulo 4, which was 17. Therefore, the answer is **C**.

7. On the most recent exam in Prof. Xochi's class,

- 5 students earned a score of at least 95%,
- 13 students earned a score of at least 90%,
- 27 students earned a score of at least 85%, and
- 50 students earned a score of at least 80%.

How many students earned a score of at least 80% and less than 90%?

A 8

B 14

C 22

D 37

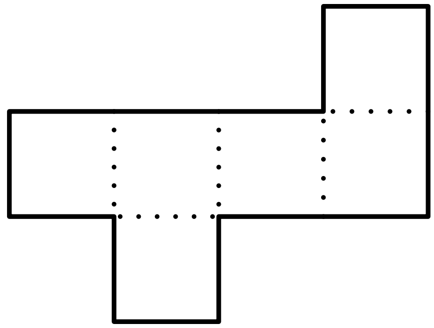
E 45

Solution:

Notice that the numbers of students in each category keeps increasing, which makes sense because the categories are getting broader.

We want to include all 50 of the students who earned a score of at least 80% but exclude all 13 of the students who earned a score of at least 90%. So, the answer is $50 - 13 = 37$, which is choice **D**.

8. Isaiah cuts open a cardboard cube along some of its edges to form the flat shape shown, which has an area of 18 square centimeters. What was the volume of the cube in cubic centimeters?



A $3\sqrt{3}$

B 6

C 9

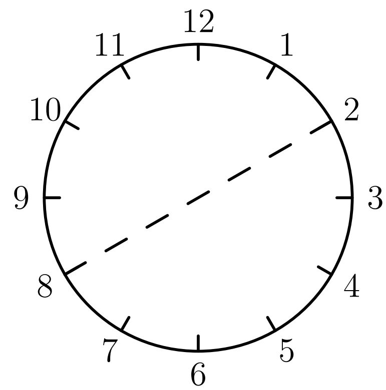
D $6\sqrt{3}$

E $9\sqrt{3}$

Solution:

There are 6 squares, so each has area $18 \div 6 = 3$. Then the side length of the cube is $\sqrt{3}$. The volume is $\sqrt{3} \times \sqrt{3} \times \sqrt{3} = 3\sqrt{3}$, which is choice **A**.

9. Ningli looks at the 6 pairs of numbers directly across from each other on a clock. She takes the average of each pair of numbers. What is the average of the resulting 6 numbers?



A 5

B 6.5

C 8

D 9.5

E 12

Solution:

The answer is the same as the average of all 12 numbers. To understand why this is the case, it is actually generally true that if a whole bunch of numbers is split up into pairs, and each pair is averaged, and then all those pair-averages are averaged, the answer is the average of all the numbers.

To see why this is true, consider a smaller example of 6 numbers a, b, c, d, e, and f. The average of the pair-averages is:

$$\frac{1}{3}\left(\frac{a+b}{2}+\frac{c+d}{2}+\frac{e+f}{2}\right)$$

and that is equal to

$$\frac{a+b+c+d+e+f}{6}.$$

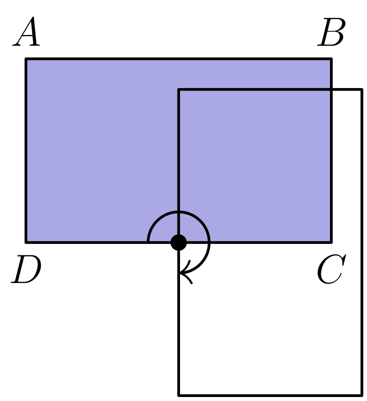
The same type of simplification happens with 12 numbers.

Since the 12 numbers are equally spaced (in an arithmetic progression), the answer is the same as the average of the first and last number, which is

$$\frac{1+12}{2} = \frac{13}{2} = 6.5,$$

or choice **B**.

10. In the figure below, $ABCD$ is a rectangle with sides of length $AB = 5$ inches and $AD = 3$ inches. Rectangle $ABCD$ is rotated 90° clockwise around the midpoint of side DC to give a second rectangle. What is the total area, in square inches, covered by the two overlapping rectangles?



- A 21
- B 22.25
- C 23
- D 23.75
- E 25

Solution:

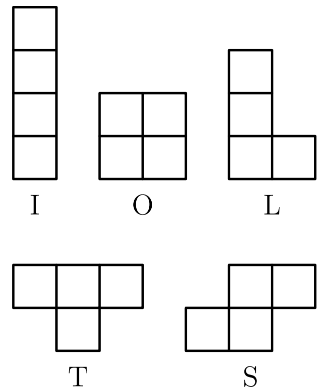
The easiest way to solve this problem is using the Inclusion-Exclusion formula. That is: add the areas of the two rectangles, and then subtract the overlapping (square) area.

Each rectangle has area $5 \times 3 = 15$.

Their overlap is a square that has side length 2.5, and so its area is $2.5^2 = 6.25$.

Therefore, the total area is $15 + 15 - 6.25 = 23.75$, which is choice **D**.

11. A *tetromino* consists of four squares connected along their edges. There are five possible tetromino shapes, I, O, L, T, and S, shown below, which can be rotated or flipped over. Three tetrominoes are used to completely cover a 3×4 rectangle. At least one of the tiles is an S tile. What are the other two tiles?



A I and L

B I and T

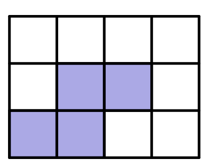
C L and L

D L and S

E O and T

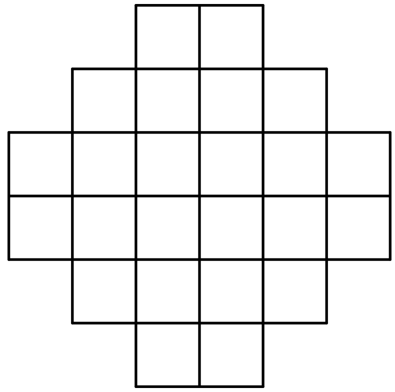
Solution:

A little trial-and-error suggests placing the S tetromino in a way that does not block too much, like this:



It is then easy to see that the remainder can be partitioned into two L tetrominoes, and so the answer is **C**.

12. The region shown below consists of 24 squares, each with side length 1 centimeter. What is the area, in square centimeters, of the largest circle that can fit inside the region, possibly touching the boundaries?



A 3π

B 4π

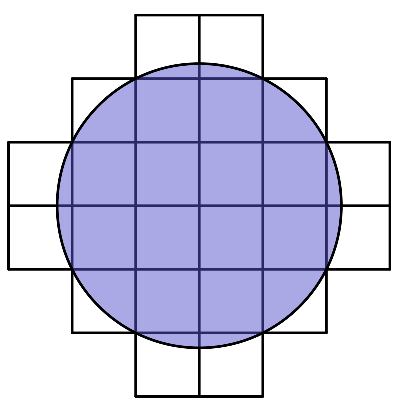
C 5π

D 6π

E 8π

Solution:

The corners of the region which are closest to the center are the 8 points which lie on this circle:



By the Pythagorean Theorem, each of these points has this distance from the center:

$$\sqrt{1^2 + 2^2} = \sqrt{5}.$$

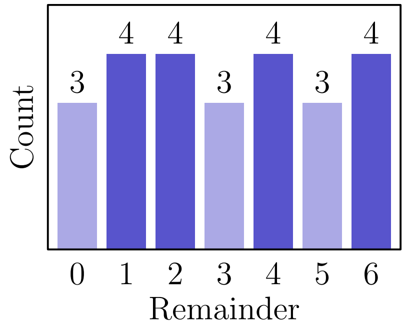
The area of the circle is then

$$\pi(\sqrt{5})^2 = 5\pi,$$

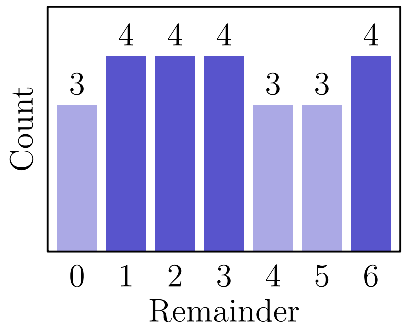
which is choice **C**.

13. Each of the even numbers $2, 4, 6, \ldots, 50$ is divided by 7. The remainders are recorded. Which histogram displays the number of times each remainder occurs?

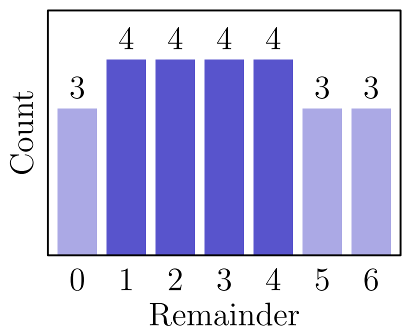
A



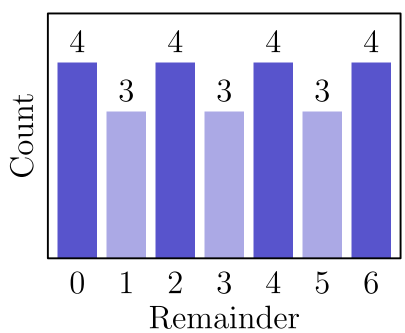
B



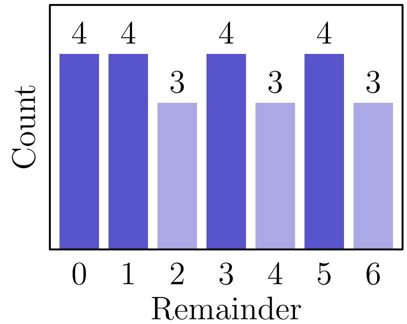
C



D



E



Solution:

The remainders go in order $2, 4, 6, 1, 3, 5, 0$, and then repeat.

All of the answer choices have 3 bars of height 3 and 4 bars of height 4.

So, we should pick the answer choice where the taller bars are on the first remainders to appear in our order, which are $2, 4, 6$, and 1. That is option **A**.

14. A number N is inserted into the list $2, 6, 7, 7, 28$. The mean is now twice as great as the median. What is N?

A 7

B 14

C 20

D 28

E 34

Solution:

After inserting N into the list, there will be 6 total numbers. That is even, so the median will be the average of the middle two numbers.

All of the answer choices are at least 7, so when they are inserted into the list, the middle two numbers will be 7 and 7. It is convenient that the median will always be 7, no matter which answer choice is picked.

The mean becomes twice the median, which is $2 \times 7 = 14$.

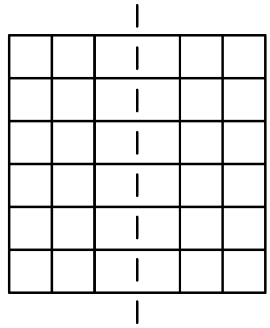
To have a total of 6 numbers with mean 14, their sum must become $6 \times 14 = 84$.

The sum of the original 5 numbers is

$$2 + 6 + 7 + 7 + 28 = 50,$$

so the answer is $84 - 50 = 34$, which is choice **E**.

15. Kei draws a 6-by-6 grid. He colors 13 of the unit squares silver and the remaining squares gold. Kei then folds the grid in half vertically, forming pairs of overlapping unit squares. Let m and M equal the least and greatest possible number of gold-on-gold pairs, respectively. What is the value of $m + M$?



A 12

B 14

C 16

D 18

E 20

Solution:

The number of gold squares is

$$6 \times 6 - 13 = 36 - 13 = 23.$$

The 36 total squares overlap as 18 pairs.

To minimize the number of pairs with two gold squares, the gold squares should first be spread out across all pairs. That uses up 18 of them. The remaining $23 - 18 = 5$ gold squares double-up and create a total of $m = 5$ gold-on-gold pairs.

To maximize the number of pairs with two gold squares, the 23 gold squares should first be paired up as much as possible. That can be done to create $M = 11$ pairs, with 1 gold square left over, because $23 \div 2$ is 11 with a remainder of 1.

The answer is $m + M = 5 + 11 = 16$, which is choice **C**.

16. Five distinct integers from 1 to 10 are chosen, and five distinct integers from 11 to 20 are chosen. No two numbers differ by exactly 10. What is the sum of the ten chosen numbers?

A 95

B 100

C 105

D 110

E 115

Solution:

Call the integers from 1 to 10 inclusive the *lower range*, and call the integers from 11 to 20 inclusive the *higher range*.

Each of the 5 distinct numbers chosen from the lower range blocks out the number in the higher range that is exactly 10 more than itself. There are only 10 numbers in the higher range, so there are only $10 - 5 = 5$ numbers not yet blocked.

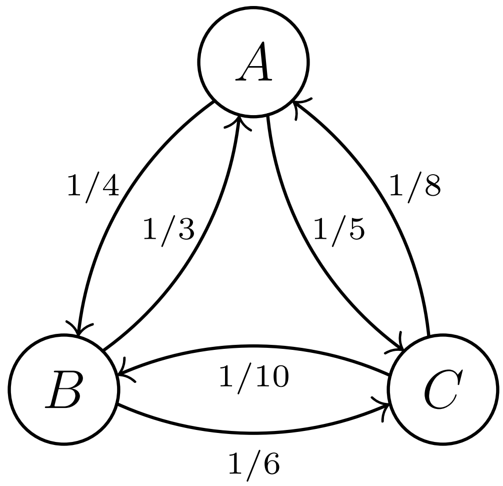
We need to choose 5 distinct numbers from the higher range, so the numbers chosen from the higher range are precisely those which are not yet blocked. They are each exactly 10 more than a not-chosen number in the lower range.

So, the sum of the 5 distinct numbers chosen from the higher range is exactly $5 \times 10 = 50$ more than the sum of the 5 not-chosen numbers in the lower range.

The sum of all 10 chosen numbers is therefore equal to 50 plus the sum of all chosen and not-chosen numbers in the lower range $1 + 2 + \cdots + 10$.

The sum of the numbers from 1 to 10 is $\frac{10(10+1)}{2} = 55$, so the answer is $50 + 55 = 105$, or choice **C**.

17. In the land of Markovia, there are three cities: A, B, and C. There are 100 people who live in A, 120 who live in B, and 160 who live in C. Everyone works in one of the three cities, and a person may work in the same city where they live. In the figure below, an arrow pointing from one city to another is labeled with the fraction of people living in the first city who work in the second city. (For example, $\frac{1}{4}$ of the people who live in A work in B.) How many people work in A?



A 55

B 60

C 85

D 115

E 160

Solution:

The number of people who live in A and work in A is

$$100 - 100 \times \frac{1}{4} - 100 \times \frac{1}{5}$$

which is

$$100 - 25 - 20 = 55.$$

The number of people who live in B and work in A is

$$120 \times \frac{1}{3} = 40$$

The number of people who live in C and work in A is

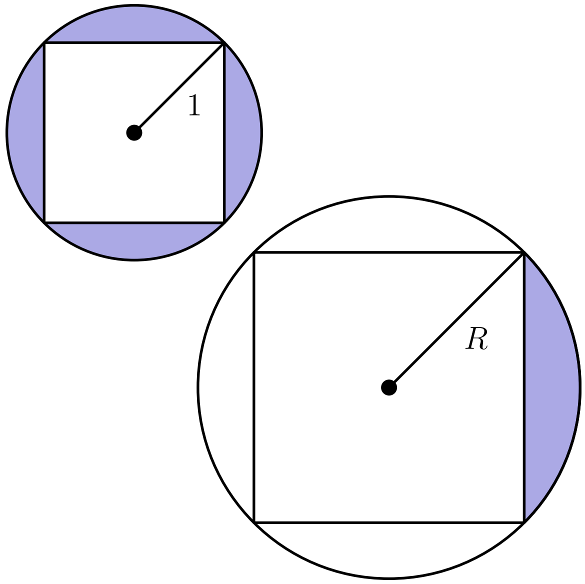
$$160 \times \frac{1}{8} = 20$$

So, the answer is

$$55 + 40 + 20 = 115,$$

which is choice **D**.

18. The circle shown below on the left has a radius of 1 unit. The region between the circle and the inscribed square is shaded. In the circle shown on the right, one quarter of the region between the circle and the inscribed square is shaded. The shaded regions in the two circles have the same area. What is the radius R, in units, of the circle on the right?



A $\sqrt{2}$

B 2

C $2\sqrt{2}$

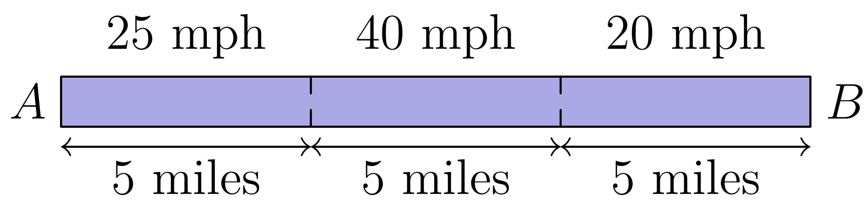
D 4

E $4\sqrt{2}$

Solution:

The diagram on the right is similar to the diagram on the left, but the corresponding areas in the diagram on the right are 4 times the areas on the left. So, each length on the right is $sqrt4 = 2$ times the corresponding length on the left. This gives $R = 2$, which is choice **B**.

19. Two towns, A and B, are connected by a straight road, 15 miles long. Traveling from town A to town B, the speed limit changes every 5 miles: from 25 to 40 to 20 miles per hour (mph). Two cars, one at town A and one at town B, start moving toward each other at the same time. They drive at exactly the speed limit in each portion of the road. How far from town A, in miles, will the two cars meet?



A 7.75

B 8

C 8.25

D 8.5

E 8.75

Solution:

Think of the road as having three sections: left, middle, and right. Each section is 5 miles long.

The car from A reaches the middle section in

$$\frac{5}{25} = \frac{1}{5}$$

hours.

The car from B reaches the middle section in

$$\frac{5}{20} = \frac{1}{4}$$

hours. By that time, the car from A has already driven in the middle section for

$$\frac{1}{4} - \frac{1}{5} = \frac{1}{20}$$

hours. During this time, that car from A has traveled

$$40 \times \frac{1}{20} = 2$$

miles in the middle section.

That leaves $5 - 2 = 3$ miles between the two cars in the middle section.

At that moment, the car from A is $5 + 2 = 7$ miles from A.

Since the cars drive at the same speed of 40 mph in the middle section, they meet after each driving 1.5 more miles. This takes the car from A a total distance of $7 + 1.5 = 8.5$ miles, which is choice **D**.

20. Sarika, Dev, and Rajiv are sharing a large block of cheese. They take turns cutting off half of what remains and eating it: first Sarika eats half of the cheese, then Dev eats half of the remaining half, then Rajiv eats half of what remains, then back to Sarika, and so on. They stop when the cheese is too small to see. About what fraction of the original block of cheese does Sarika eat in total?

A $\frac{4}{7}$

B $\frac{3}{5}$

C $\frac{2}{3}$

D $\frac{3}{4}$

E $\frac{7}{8}$

Solution:

Sarika gets $\frac{1}{2}$ of the cheese in the first step.

Then Dev gets $\frac{1}{2^2}$ of the cheese.

Then Rajiv gets $\frac{1}{2^3}$ of the cheese.

Sarika then gets another $\frac{1}{2^4}$ of the cheese.

This pattern continues. Ultimately, Sarika gets

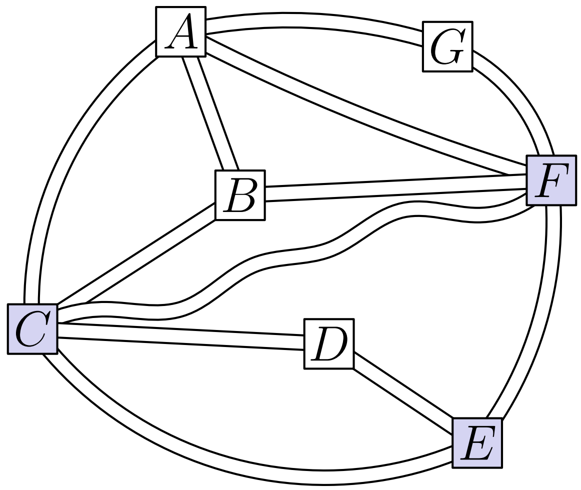
$$\frac{1}{2}+\frac{1}{2^4}+\frac{1}{2^7}+\dots$$

which is the sum of an infinite geometric series with first term $a=\frac{1}{2}$ and common ratio $r=\frac{1}{2^3}$. That sum is

$$\frac{1}{1-r}=\frac{\frac{1}{2}}{1-\frac{1}{8}}=\frac{\frac{1}{2}}{\frac{7}{8}}=\frac{4}{7}$$

which is choice **A**.

21. The Konigsberg School has assigned grades 1 through 7 to pods A through G, one grade per pod. Some of the pods are connected by walkways, as shown in the figure below. The school noticed that each pair of connected pods has been assigned grades differing by 2 or more grade levels. (For example, grades 1 and 2 will not be in pods directly connected by a walkway.) What is the sum of the grade levels assigned to pods C, E, and F?



A 12

B 13

C 14

D 15

E 16

Solution:

Among pods A, B, C, and F, each pair is directly connected by a walkway. So, they have to be at least 2 grades apart. The only way to select 4 numbers from 1 through 7 inclusive, which are at least 2 apart, is to select $\{1, 3, 5, 7\}$ in some order.

Try dealing with G next, because it looks less complex. What if grade 2 is in pod G?

Then A and F cannot be 1 or 3. That forces 1 and 3 to be among B and C, and 5 and 7 to be among A and F.

The remaining undetermined pods D and E only are directly connected to C and F, so this suggests putting grade 1 in pod C and grade 7 in pod F, as the extreme grades only have one conflict each.

Then grade 6 can go in pod D and grade 4 can go in pod E.

In order to satisfy the constraints we have made so far, grade 3 goes in pod B, and grade 5 goes in pod A. This actually works, and so the answer is $1+4+7=12$, which is choice **A**.

22. A classroom has a row of 35 coat hooks. Paulina likes coats to be equally spaced, so that there is the same number of empty hooks before the first coat, after the last coat, and between every coat and the next one. Suppose there is at least 1 coat and at least 1 empty hook. How many different numbers of coats can satisfy Paulina's pattern?

A 2

B 4

C 5

D 7

E 9

Solution:

Imagine adding an extra coat hook with a coat on it after the last coat. Now, there will be 36 coat hooks, and a repeating pattern, where each block of the pattern has a bunch of empty hooks followed by a coat. Let b be the number of items in each block. Let d be the number of blocks. Note that d is exactly one more than the number of coats, because we added an extra coat at the end.

We then have $bd = 36$.

The constraint on b is that $b \geq 2$ because each block has at least one empty hook, and ends with a coat.

The constraint on d is that $d \geq 2$ because there was at least one coat before, and we added one extra coat at the end.

So, we just need to find out how many ways there are to factorize 36 into the product of two integers that are at least 2.

The number of ways to factorize 36 into the product of two positive integers is exactly equal to the number of factors of 36. There is a formula for that: since $36 = 2^2 \times 3^2$, the number of factors of 36 is

$$(2+1)(2+1) = 9.$$

Out of these factorizations, exactly two are disqualified: 1×36 and 36×1. So, the answer is $9 - 2 = 7$, which is choice **D**.

23. How many four-digit numbers have all three of the following properties?

(I) The tens digit and ones digit are both 9.

(II) The number is 1 less than a perfect square.

(III) The number is the product of exactly two prime numbers.

A 0

B 1

C 2

D 3

E 4

Solution:

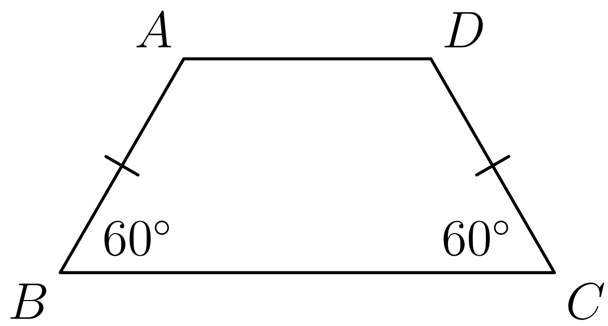
The number has the form $XX99$, and so the perfect square that is 1 more ends in 00, and so it is the square of a number ending in 0. Suppose that square is a^2.

In order for $a^2 - 1$ to be a 4-digit number, the only possibilities for a are $\{4, 5, \ldots, 10\}$.

Since $a^2 - 1 = (a-1)(a+1)$, we also need both $a - 1$ and $a + 1$ to be prime. We are then looking for pairs of prime numbers that are right around $\{40, 50, \ldots, 100\}$.

Going through all the possibilities, the only ones that work are 59 and 61, and so there is exactly 1 way to do this. The answer is **B**.

24. In trapezoid $ABCD$, angles B and C measure 60° and $AB = DC$. The side lengths are all positive integers and the perimeter of $ABCD$ is 30 units. How many non-congruent trapezoids satisfy all of these conditions?



A 0

B 1

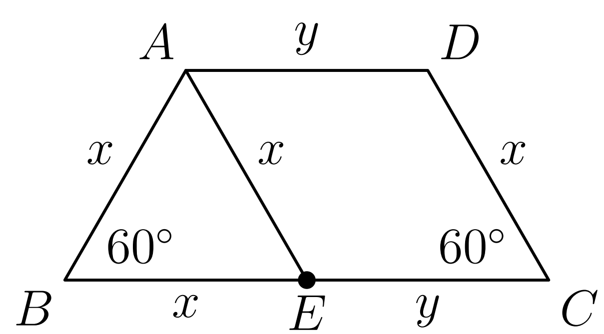
C 2

D 3

E 4

Solution:

Draw a line through A parallel to line CD, and observe that some lengths are automatically equal to each other:



That is because $\angle AEB = \angle DCB = 60^\circ$, and so triangle ABE is equilateral. All lengths labeled x are always equal.

Also, $ADCE$ is a parallelogram (not necessarily a rhombus), so the remaining lengths labeled y are always equal to each other, but not necessarily equal to x.

The perimeter is $3x + 2y$, but it is also supposed to be 30.

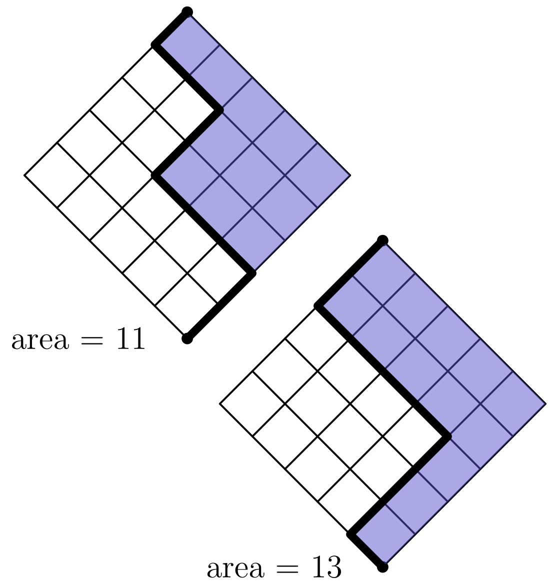
The problem the amounts to counting how many positive integer solutions there are to the equation $3x + 2y = 30$.

As we consider positive integers for x, observe that it is precisely the even integers which give an integer solution for y, because $30 - 3x$ is the even number $2y$.

And, once we get to $x = 10$, the y that satisfies the equation is $y = \frac{30-3\times 10}{2} = 0$, which is not a positive integer.

So, the values that work for x are the positive even integers less than 10, or $\{2, 4, 6, 8\}$. There are 4 options, which gives the answer of **E**.

25. Makayla finds all the possible ways to draw a path in a 5×5 diamond-shaped grid. Each path starts at the bottom of the grid and ends at the top, always moving one unit northeast or northwest. She computes the area of the region between each path and the right side of the grid. Two examples are shown in the figures below. What is the sum of the areas determined by all possible paths?



A 2520

B 3150

C 3840

D 4730

E 5050

Solution:

Let X be the answer.

By symmetry, if the question asked for the sum of areas between each path and the **left** side of the grid, then the answer would be exactly the same X.

But if that answer is added to the original answer, that is exactly the same as the sum over all paths, of the sum of areas to the left and to the right.

For each path, that sum of areas is exactly 25.

The number of paths is equal to the number of ways to rearrange $LLLLLRRRRR$, where L stands for Left and R stands for Right, as the path walks up. The number of rearrangements is 10 choose 5, denoted $\binom{10}{5}$.

So, $2X = 25 \times \binom{10}{5}$. Dividing by 2, we get that X equals

$$\frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} \times \frac{25}{2},$$

which is 3150, or choice **B**.

Problems: https://live.poshenloh.com/past-contests/amc8/2025